

His - Book' 1771.



His - Book' 1771.



THE
COMPLETE MEASURER:
OR, THE
Whole Art of Measuring.

IN TWO PARTS.

The First PART teaching
DECIMAL ARITHMETICK, with the
Extraction of the SQUARE and CUBE ROOTS:
And also the *Multiplication of Feet and Inches*, commonly
called CROSS MULTIPLICATION.

The Second PART teaching to
Measure all Sorts of SUPERFICIES and SOLIDS,
by *Decimals*; by *Cross Multiplication*, and by *Scale and*
Compasses: Also the Works of several Artificers,
relating to *Building*; and the Measuring of *Board and*
Timber. Shewing the common Errors.

And some *Practical* QUESTIONS.

The *Thirteenth* EDITION. To which is added,
An APPENDIX. 1. Of Gauging. 2. Of Land-Measuring.

Very useful for all Tradesmen; especially Carpenters, Bricklayers,
Plasterers, Painters, Joiners, Glaziers, Masons, &c.

By WILLIAM HAWNEY, *Philomath.*

Recommended by the Rev. Dr. John Harris, F. R. S.

L O N D O N:

Printed for H. WOODFALL, J. and F. RIVINGTON,
R. BALDWIN, L. HAWES and W. CLARKE and R.
COLLINS, G. KEITH, S. CROWDER, T. LONGMAN,
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ROBINSON and J. ROBERTS. 1769.

I Have perused this **BOOK**, and recommend it to the Publick as a very useful One.

J. HARRIS, D. D.



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easy Method how most of what is Necessary
and Useful in this Science, may be under-
stood. Translated by the above-mentioned
Dr. Harris, and referred to in the Preface
of this Book.



T·H·E
P R E F A C E.

HAVING perused several Books concerning the Mensuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book so perfect, as to give any tolerable Satisfaction to a Learner; and I having practised and taught Measuring for several Years, and thereby gained Experience and Knowledge in that Art, having learned some Things from one Author, and some Things from another, I began to think of digesting my Thoughts into some such Method as might give a Learner full Satisfaction, without being at the Charge of buying so many Books; and being importuned thereunto by some Friends, I fell to work, and at last brought them to that Perfection you here find in the following Work.

1. As to the *Decimal Arithmetick*, I have been as brief as the Matter would well bear, to make it plain.

A 2

2. As

iv . P R E F A C E .

2. As to the Multiplying of Feet and Inches, commonly called *Cross Multiplication*, my Method differs from that which is usually taught in other Authors, as being (I think) much shorter and plainer.

3. In measuring of Superficies and Solids, I have given the Demonstration of the Rules, which I thought might be very acceptable to the Ingenious; for, indeed, I always look upon the Writing of a Rule without a Demonstration (in any Part of the Mathematicks) to be but lame and defective; and for want of knowing the Reason of the Rule, a Learner may commit great Errors; besides, when a Learner knows the Reason of the Rules, he may retain them better in his Memory. The Rule for measuring a Prismoid and Cyliindroid, I had out of Mr. *Everard's Art of Gauging*; but the Reason he does not shew, neither have I found it in any other Author; but that the Method is true, I have endeavoured to make plain.

The Demonstration of the Rules for finding the Area of an Ellipsis and Parabola; also the Demonstration of the Rules for finding the solid Content of the Frustum of a Cone and Pyramid, the Solidity of a Globe of a Spheroid, a Parabolic Conoid, and of a Parabolic Spindle, and their Frustums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide*; where the curious and ingenious Reader may see many other Demonstrations algebraically performed. I have also demonstrated the Rule for finding the Solidity of a Globe, out of *Pardie's Elements of Geometry* (Book the 5th, Art. the 33d) published in *English* with many
Additions

P R E F A C E.

Additions, by the Reverend Dr. *Harris*, F. R. S. and the same is also done out of *Sturmius's Mathesis Enucleata*; so that the ingenious Reader may use which of those Ways he likes best.

The Scale supposed to be used in all the Operations, is the Line of Numbers, commonly called *Gunter's Line*, which is upon the ordinary Two-Foot or Eighteen-Inch Rules, commonly used by the Carpenters, Masons, &c. because I thought it needless, as well as impertinent, to write the Use of Sliding-Rules, or any other particular Scales, they being sufficiently treated of by several Authors; viz. by the above-named Mr. *Everard*, in his *Art of Gauging* above-mentioned, where you have the Use of a Sliding-Rule in Arithmetick, Geometry, in Measuring of Superficies and Solids, Gauging, &c. Likewise Mr. *Hunt* has written largely of the Uses of his Sliding-Rule, in Arithmetick, Geometry, Trigonometry, Gauging, Dialling, &c. There are several others who have explained the Use of their own Rules; so that the more curious Readers may find full Satisfaction in those Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inserted in this Place; that is, how to find a Number upon the Line. If the Number you would find consists only of Units, then the Figures upon the Line represent the Number sought: Thus, if the Number be 1, 2, 3, &c. then 1, 2, 3, &c. upon the Line, represents the Number sought. But if the Number consists of two Figures, that is, of Units and Tens, then the Figure upon the Rule stands for

the Tens, and the large Divisions stand for the Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Divisions (counted forwards) is the Point representing 34; and if 340 were to be found, it will be at the same Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and four of the smaller Divisions (counted forward) is the Point representing 304. If the Number consists of four Places, or Thousands, then the Figure upon the Line stands for Thousands, and the larger Divisions are Hundreds, the lesser Divisions are Tens, and the tenth Parts of those lesser Divisions are Units. Thus, if 2735 were to be found, then the 2 is 2000; and the 7 larger Divisions (counted forward) is 700 more; and 3 of the lesser Divisions is 30 more; and half of one of the lesser Divisions is 5 more, which is the Point representing 2735. You must remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Divisions; and each of those larger Divisions are subdivided (or supposed so to be) into 10 other Parts, which I call the smaller Divisions; and each of those Parts supposed to be subdivided again into ten other Parts, &c. You must also remember, that if 1 in the Middle of the Line stands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End signifies 1, then 1 in the Middle stands for 10, and 1 at the upper End is 100, &c.

There is one Thing more which I would have my Reader to understand; and that is, how to find
all

all such proportional Numbers made use of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Use to know how to correct a Number, which may happen to be false printed, or to enlarge any Number to more decimal Places, for more Exactness; for though I have mentioned what such Numbers are, yet I have not shewn how to find them, which a Learner may be a little at a Nonplus to do; though they are easily found by the Rules there laid down. I shall therefore give two or three Examples, in this Place, of finding such Numbers, which may enable my Reader to find out the rest.

And, first, let it be required to find the Area of a Circle, whose Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1416926, &c. whereof 3.1416 is sufficient in most Cases. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by 5 (*viz.* half 3.1416 by half 1) and the Product is .7854, which is the Area of the Circle, whose Diameter is 1.

Again; if the Area be required when the Circumference is 1, first find what the Diameter will be, thus: 3.1416 : to 1 :: so is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is .159154 by .5, and the Product is .079577, which is the Area of a Circle whose Circumference is 1.

If

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7854 is .88.62, which is the Side of a Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whose Circumference is 1.

If the Side of a Square within a Circle be required, if you square the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that shall be the Side of the Square which may be inscribed in that Circle; so if the Diameter of the Circle be 1, then the half is .5; which squared, is .25; and this, doubled, is 5, whose Square Root is .7071, the Side of the Square inscribed.

Again; If the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonstrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the same Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be also 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe required. Now if the Diameter be 1, the Area of the Circle, or Base of the Cylinder, is .7854 (as is above shewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is also .7854, the Solidity of the Cylinder; $\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe, whose Diameter is 1.

From

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From what has been said, the Reader may easily perceive how all other proportional Numbers are found, and may examine them at his Pleasure.

I shall not enlarge any farther upon the Matter, but leave the Book to speak for itself; and if it prove beneficial to the ingenious Practitioners, I have my Desire. So, wishing my ingenious Reader good Success in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Desire of his Well-wisher,

W. HAWNEY.



T H E



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THE



THE Complete MEASURER.

PART I.

CHAP. I.

Notation of DECIMALS.



DECIMAL Fraction is an artificial Way of setting down and expressing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annexed to it, and must therefore be either 10, 100, 1000, 10000, &c. and therefore in writing down of a Decimal Fraction, there is no Necessity of writing down the Denominator; for by bare Inspection it is certainly known, consisting of an Unit with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

B

Example.

merator, they must be supplied by prefixing so many Cyphers before the Figures of your Numerator; as, suppose $\frac{19}{1000}$ were to be written down, without its Denominator; here, because there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and set it down thus, .019.

The Integers are separated from the Decimals several Ways, according to Mens Fancies; but the best and most usual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is sufficient: Thus, if you were to write down $317\frac{217}{1000}$ it may be thus expressed, 317.217; and $59\frac{25}{1000}$ thus, 59.0025; and $\frac{75}{1000}$ thus, .0075, &c.



CHAP II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Cases: 1st, To reduce a Vulgar Fraction to a Decimal. 2^{dly}, To find the Value of a Decimal in the known Parts of Coin, Weights, Measures, &c. 3^{dly}, To reduce Coin, Weights, Measures, &c. to a Decimal. Of these in their Order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, so is an Unit (with a competent Number of Cyphers annexed) to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Re-

sult by the Denominator, the Quotient is the Decimal equivalent to the Vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduced to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal required, and is equivalent to $\frac{3}{4}$ given.

Note, That so many Cyphers as you annex to the given Numerator, so many Places must be pricked off in the Decimal found; and if it should happen, that there are not so many Places of Figures in the Quotient, the Deficiency must be supplied, by prefixing so many Cyphers before the Quotient Figures, as in the next Example.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having six Places.

To the Numerator annex six Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was required to have six Places, therefore you must prefix two Cyphers before it, and then it will be 005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the Work of these two Examples.

$$\begin{array}{r} 4 \overline{) 3.006.75} \\ \underline{28} \\ 20 \\ \underline{20} \\ .. \end{array}$$

$$\begin{array}{r} 573 \overline{) 3.000000(005235} \\ \underline{0000} \\ 1350 \\ \underline{2040} \\ 3210 \\ \underline{0000} \\ 345 \end{array}$$

In the second Example there remains 345, which Remainder is very insignificant, it being less than $\frac{1}{100000}$ Part of an Unit, and therefore is rejected.

II. To

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Measures, &c.*

The R U L E.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off so many Places to the Right-hand as there were Places in the Decimal given; and multiply those Figures pricked off by the Number of Parts in the next inferior Denomination, and prick off so many Places as before, and so continue to do, till you have brought it to the lowest Denomination required.

Example 1. Let .7565 of a Pound Sterling be given to be reduced to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and by 4, as the Rule directs, and always prick off four Places to the Right-hand, and you will find it make 15s. 1d. 2q. See the Work.

$$\begin{array}{r}
 .7565 \\
 \times 20 \\
 \hline
 15.1300 \\
 \times 12 \\
 \hline
 1.5600 \\
 \times 4 \\
 \hline
 2.2400
 \end{array}$$

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the first Figure, (or Place of Primes) and it makes so many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then

B 3

for

6 *Reduction of DECIMALS. Part I.*

for every Unit in the second Place count ten, and to that add the Figure in the third Place, and reckon that so many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

First, Double your 6, and it makes it 12 s. then take 5 out of 9, and for that reckon another Shilling, and it makes 13 s. and the 4 remaining is four Tens, and the 5 makes 45, which being above 38, you must therefore cast away 2, and there rest 43 Farthings, which is 10 d. $\frac{3}{4}$. So the Answer is 13 s. 10 d. $\frac{3}{4}$.

| | l. | s. | d. |
|-----------------------|------|----|---------------|
| So the Value of .725 | = 14 | 6 | |
| And the Value of .878 | = 17 | 6 | $\frac{3}{4}$ |
| And the Value of .417 | = 8 | 4 | |

And so of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right-hand, and you will find the Answer to be 7 oz. 3 pwt. 13 gr. *ferè*. See the Work.

| | |
|---------|-----------------|
| .59755 | |
| 12 | |
| 7.17060 | |
| 20 | |
| 143.412 | oz. pwt. gr. |
| 24 | Facit 7 3 9.888 |
| 164800 | |
| 82400 | |
| 9.88800 | |

Let

Chap. 2. *Reduction of DECIMALS.* 7

Let .43569 of a Ton be reduced to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Answer will be 8 C. 2 qrs. 24 lb. *ferè.*

$$\begin{array}{r}
 .43569 \\
 \underline{20} \\
 8.71380 \\
 \underline{4} \quad \text{C. qrs. lb.} \\
 \text{Facit } 8 \quad 2 \quad 23.9456 \\
 2.85520 \\
 \underline{28} \\
 23.94560
 \end{array}$$

Let .9595 of a Foot be reduced into Inches and Quarters.

$$\begin{array}{r}
 .9595 \\
 \underline{12} \\
 11.5140 \\
 \underline{4} \quad \text{Facit } 11 \text{ Inches, } 2 \text{ Quarters.} \\
 2 \text{ } 0560
 \end{array}$$

III. *To reduce the known Parts of Money, Weight, Measure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the lesser Denomination given, annex a competent Number of Cyphers, and divide by the Number of such Parts that are contained in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal sought.

Example

Example 1. Let 6 *d.* be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers (suppose 3), and divide the Result by 240 (the Pence in a Pound), and the Quotient is the Decimal required.

$$240 \overline{) 6.000} (.025$$

$$\begin{array}{r} 1200 \\ \hline \text{Facit } .025 \end{array}$$

....

Example 2. Let 3 *d.* $\frac{3}{4}$ be reduced to the Decimal of a Pound, having six Places.

In 3 *d.* $\frac{3}{4}$ there are fifteen Farthings, therefore to 15 annex six Cyphers (because there are to be six Places in the Decimal required), and divide by 960 (the Farthings in a Pound), and the Quotient is .015625.

$$960 \overline{) 15.000000} (.015625$$

.....

$$\begin{array}{r} 540 \\ 600 \\ 240 \\ 480 \\ \hline \end{array}$$

...

Example 3. Let 3 $\frac{1}{4}$ Inches be reduced to the Decimal of a Foot, consisting of four Places.

In 3 $\frac{1}{4}$ Inches, there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48 (the Quarters in a Foot), and the Quotient is .2708.

$$48 \overline{) 13.0000} (.2708$$

$$\begin{array}{r} 340 \\ 400 \\ \hline 16 \end{array}$$

Example

Chap. 3. *Addition of DECIMALS.* 9

Example 4. Let 9 C. 1 gr. 16 lb. be reduced to the Decimal of a Ton, having six Places.

| | |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>C. qu. lb.</p> <p>9 1 16</p> <hr style="width: 10%; margin-left: 0;"/> <p>4</p> <hr style="width: 10%; margin-left: 0;"/> <p>37 ^{qrs.}</p> <p>28</p> <hr style="width: 10%; margin-left: 0;"/> <p>302 <i>Facit</i> .469642</p> <p>75</p> <hr style="width: 10%; margin-left: 0;"/> <p>1052 Pounds.</p> | <p>2240)1052.000000 0(469642</p> <p>.....</p> <hr style="width: 50%; margin-left: auto; margin-right: 0;"/> <p>15600</p> <p>21600</p> <p>14400</p> <p>96000</p> <p>6400</p> <hr style="width: 50%; margin-left: auto; margin-right: 0;"/> <p>1920</p> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



C H A P. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the same Way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, &c.

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

$$\begin{array}{r}
 317.25 \\
 17.125 \\
 275.5 \\
 47.3579 \\
 12.75 \\
 \hline
 \end{array}$$

Sum 669.9829

This is so plain, that more Examples I think needless.

C H A P.

C H A P. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is performed likewise the same Way as in whole Numbers, respect being had to the right placing the Numbers (as in Addition), as in the following Examples.

(1)

From 212.0137
 Subtr. 31.1275

 Rems 180.8862

 Proof 212.0137

(2)

From 201.1250
 Subtr. 5.5785

 Rems 195.5465

 Proof 201.1250

(3)

From 2051.315
 Subtr. 79.172

 Rems 197.2143

 Proof 2051.315

(4)

From 305
 Subtr. 7.2597

 Rems 23.2403

 Proof 30.5

Note, If the Number of Places in the Decimals be more in that which is to be subtracted, than in that which you subtract from, you must suppose Cyphers to make up the Number of Places, as in the fourth Example.

C H A P. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is also performed the same Way as Multiplication of whole Numbers; but to know the Value of the Product, observe this Rule.

Cut off, or separate by a Comma or Prick, so many decimal Places in the Product, as there are Places of Decimals in both Factors, *viz.* in the Multiplicand and Multiplier; which I shall further explain in the following Examples.

Let 3.125 be multiplied by 2.75; multiply the Numbers together; as if they were whole Numbers, and the Product is 859375: And because there were three Places of Decimals pricked off in the Multiplicand, and two Places in the Multiplier, therefore you must prick off five Places of Decimals in the Product, as you may see by the Work.

$$\begin{array}{r}
 3.125 \\
 2.75 \\
 \hline
 15625 \\
 21875 \\
 6250 \\
 \hline
 8.59375
 \end{array}$$

Let

12 Multiplication of Decimals. Part I.

Let 79.25 be multiplied by .459.

In this Example, because two Places of Decimals are pricked off in the Multiplicand, and three in the Multiplier, therefore there must be five pricked off in the Product.

$$\begin{array}{r}
 79.25 \\
 \times .459 \\
 \hline
 71325 \\
 39625 \\
 31700 \\
 \hline
 36.37575
 \end{array}$$

Let .135272 be multiplied by .00425.

In this Example, because in the Multiplicand are six decimal Places, and in the Multiplier five Places; therefore in the Product there must be eleven Places of Decimals; but when the Multiplication is finished, the Product is but 57490600, viz. only eight Places; therefore, in this Case, you must prefix three Cyphers before the Product Figures, to make up the Number of eleven Places: So the true Product will be .00057490600.

$$\begin{array}{r}
 .135272 \\
 \times .00425 \\
 \hline
 676360 \\
 270544 \\
 541088 \\
 \hline
 .00057490600
 \end{array}$$

Chap. 5. *Multiplication of DECIMALS.* 13

More Examples for Practice.

| | |
|--------------------|-------------------|
| <u>.001472</u> | <u>.017532</u> |
| <u>.1045</u> | <u>347</u> |
| 7360 | 122724 |
| 5888 | 70128 |
| 14720 | 52596 |
| <u>.0001538240</u> | <u>6.083604</u> |
| <u>279.25</u> | <u>32.0752</u> |
| <u>.445</u> | <u>.0325</u> |
| 139625 | 1603760 |
| 111700 | 641504 |
| 111700 | 962256 |
| <u>124.26625</u> | <u>1.04244400</u> |
| <u>4.443</u> | <u>20.0291</u> |
| <u>15.98</u> | <u>35.45</u> |
| 35544 | 1001455 |
| 39987 | 801164 |
| 22215 | 1001455 |
| 4443 | 600873 |
| <u>70.99914</u> | <u>710.031595</u> |
| <u>7.3564</u> | <u>.75432</u> |
| <u>.0126</u> | <u>.0356</u> |
| 441384 | 452592 |
| 147128 | 377160 |
| 73564 | 226296 |
| <u>.09269064</u> | <u>.026853792</u> |

Contracted Multiplication of Decimals.

Because in Multiplication of Decimal Parts, and mixed Numbers, there is no need to express all the Figures of the Product, but in most Places two, three, or four Places of Decimals will be sufficient; therefore, to contract the Work, observe the following

R U L E.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whose Place you intend to keep in the Product; then invert the Order of all the other Figures; that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which stands over the Figure you are then multiplying withal, and set down the first Figure of each particular Product directly one under the other: But yet a due Regard must be had to the Increase arising from the Figures on the Right hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four Places retained in the Decimals of the Product.

First, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the rest of the Figures quite contrary to the usual Way, as in the following Work: Then begin to multiply, first the 5 which is left out (only with regard to the Increase which must be carried from it); saying, 8 times 5 is 40; carry 4 in your Mind, and say, 8 times 4 is 32, and 4 I carry, is 36; set down 6, and carry 3, and proceed thro' the rest of the Figures as in common Multiplication:

Chap. 5. Contracted Multiplication. 15

Then begin to multiply with 2; saying, 2 times 4 is 8, for which I carry 1 (because it is above 5), and say, 2 times 6 is 12, and 1 that I carry is 13; set down 3, and carry 1, and proceed thro' the rest of the Figures: Then multiply with 1; saying, once 6 is 6, for which I carry 1, and say, once 8 is 8, and 1 is 9; set down 9, and proceed: Then multiply with 7; saying, 7 times 8 is 56, for which carry 6 (because it is above 55), and say, 7 times 3 is 21, and 6 that I carry is 27; set down 7, and carry 2, and proceed: Then multiply with 5; saying, 5 times 3 is 15, for which carry 2, and say, 5 times 2 is 10, and 2 I carry is 12, which set down, and add all the Products together; and the total Product will be 19.6107. See the Work.

$$\begin{array}{r} 2.38645 \\ 57128 \end{array}$$

$$19.0916$$

$$4773$$

$$239$$

$$167$$

$$12$$

$$19.6107$$

Note, That in multiplying the Figure left out every Time next the Right-hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, &c.

I have here set down the Work of the last Example, wrought by the common Way, by which you may see both the Reason and Excellency of this Way, all the Figures on the Right-hand of the Line being wholly omitted.

$$\begin{array}{r}
 2.38645 \\
 8.2175 \\
 \hline
 11 \overline{) 93225} \\
 167 \overline{) 0515} \\
 238 \overline{) 645} \\
 4772 \overline{) 90} \\
 190916 \overline{) 0} \\
 \hline
 19.6106152875
 \end{array}$$

Example 2. Let 375.13758 be multiplied by 16.7324, so that the Product may have but four Places of Decimals.

First, set 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand (because four Places of Decimals were to be pricked off), and write all the rest of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the second Figure of the Multiplier 6; saying, 6 times 8 is 48, for which I carry 5 (in respect of the 8 left out), and 6 times 5 is 30, and 5 that I carry is 35; set down 5 and carry 3, and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and say 7 times 5 is 35, for which carry 4, and say 7 times 7 is 49, and 4 I carry is 53; set down 3 under the first, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and say 3 times 7 is 21, carry 2, and say 3 times 3 is 9, and 2 I carry is 11; set down 1 and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and say 2 times 3 is 6, for which I carry 1, and say 2 times 1 is 2, and 1 I carry is 3; set down 3, and 2 times 5 is 10; set down 0, and carry 1, and proceed as before. Then begin with 4, the last Figure of the Multiplier, and say 4 times 1 is 4, for which I carry nothing, because it is less than 5: Then say 4 times

Chap. 5. *Contracted Multiplication.* 17

times 5 is 20; let down 0, and carry 2, and proceed thro' the rest of the Figures of the Multiplicand. Then add all up together, and the Product is 6276.9520. See the Work.

375.13758 the Multiplicand.

4237.61 the Multiplier reversed.

37513758 the Product with 10

22508255 the Product with 6 increased with 6×8 .

2625963 the Product with 7 increased with 7×5 .

112541 the Product with 3 increased with 3×7 .

7503 the Product with 2 increased with 2×3 .

1500 the Product with 4 increased with 0.

6276.9520 the Product required.

Let the same Example be repeated, and let only one Place in Decimals be pricked off.

375.13758 the Multiplicand.

4237.61 the Multiplier reversed.

37514 the Product by 1 with the Increase of 1×7 .

22508 the Product with 6 increased with 6×3 .

2626 the Product with 7 increased with 7×1 .

113 the Product with 3 increased with 3×5 .

7 the Product with 2 increased with 2×7 .

1 the Increase only of 4×3 .

6276.9 the Product is the same as before.

More Examples for Practice.

Multiply 395.3756 by .75642; and prick off four Places in Decimals.

395.3756 the Multiplicand.

24657. the Multiplier reversed.

2767629 the Product by 7 increased with 7×6 .

197688 the Product by 5 increased with 5×5 .

23722 the Product by 6 increased with 6×7 .

1581 the Product by 4 increased with 4×3 .

76 the Product by 2 increased with 2×5 .

299.0699 the Product required.

Let the same Example be repeated, and let there be only one Place of Decimals.

395.3756

24657.

2767 the Product by 7 increased with 7×3 .

198 the Product by 5 increased with 5×5 .

24 the Product by 6 increased with $6 \times 9 + 6 \times 5$.

2 the Increase of $4 \times 9 + 4 \times 3$.

299.1 the Product.

Characters, and their Signification.

Note, That this Mark $+$ signifies Addition; as $8+5$, that is, 8 more 5, or 8 added to 5; and $8+3+7$, denotes these Numbers are to be added into one Sum.

This Mark $-$ signifies Subtraction, as $9-4$ signifies that 4 is to be taken from 9.

This Mark \times signifies Multiplication, as 7×5 signifies that 7 is to be multiplied by 5.

This Mark \div signifies Division, as $12 \div 4$ signifies 12 is to be divided by 4.

This Mark $=$ signifies Equality, or Equation; that is, when $=$ is placed between Numbers, or Quantities, it denotes them to be equal, as $7+5=12$, that is, 7 more 5 is equal to 12; and $15-7=8$, that is, 15 less by 7, is equal to 8, or subtract 7 from 15 and there remains 8.

This Mark $::$ is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion; thus $4:20::6:30$, to be thus read, as 4 is to 20, so is 6 to 30.

C H A P. VI.

Division of DECIMALS.

DIVISION of Decimals is performed after the same Manner as Division of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty; for the resolving of which, observe either of the following

R U L E S:

R U L E S.

I. The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend which stands (or is to be supposed to stand) over the Unit's Place in the Divisor, at the first seeking.

II. When the Work of Division is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Divisor; for that Excess is the Number of Places which must be separated in the Question for Decimals: But if there be not so many Figures in the Quotient as is the said Excess, that Deficiency must be supplied with Cyphers in the Quotient, prefixed before the significant Figures thereof, towards the Left-hand, with a Point before them; so shall you plainly discover the Value of the Quotient.

These following Directions ought also to be carefully observed.

If the Divisor consists of more Places than the Dividend, there must be a competent Number of Cyphers annexed to the Dividend, to make it consist of as many (at least) or more Places of Decimals than the Divisor; for the Cyphers added must be reckoned as Decimals.

Consider whether there be as many Decimal Parts in the Dividend as there are in the Divisor; if there be not, make them so many, or more, by annexing of Cyphers.

In dividing of whole or mixed Numbers, if there be a Remainder, you may bring down more Cyphers; and, by continuing your Division, carry the Quotient to as many Places of Decimals as you please.

These Things being considered, I shall proceed to the Practice of Division of Decimals, which I shall endeavour to explain in as familiar and easy a Method as possible.

Example

Example 1. Let 48 be divided by 144.

In this Example the Divisor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers (*viz.* four), with a Point between them, and divide according to the usual Way.

$$\begin{array}{r}
 144 \overline{) 48.0000(.3333} \\
 \underline{480} \\
 480 \\
 \underline{480} \\
 48
 \end{array}$$

But, first, in seeking how often 144 in 48.0 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the first Place of Decimals; therefore the first Figure in the Quotient is in the first Place of Decimals: Or, by the second Rule, there being four Places of Decimals in the Dividend, and none in the Divisor; so the Excess of decimal Places in the Dividend, above that in the Divisor, is four; so that when the Division is ended, there must be four Places of Decimals in the Quotient. See the Work.

Example 2. Let 217.75 be divided by 65.

First, in seeking how often 65 in 217 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the Unit's Place of the Dividend; therefore the first Figure in the Quotient will be Units, and all the rest Decimals: Or, by the second Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Divisor, therefore the Excess of Decimal Places in the Dividend, above the Divisor, is two; so when the Division is ended, separate two Places in the Quotient, towards the Right-hand by a Point. See the Work.

65)

$$\begin{array}{r}
 65 \overline{) 217.75(3.35} \\
 \underline{210} \\
 77 \\
 \underline{75} \\
 25 \\
 \underline{25} \\
 00
 \end{array}$$

Example 3. Let 267.15975 be divided by 13.25

$$13.25 \overline{) 267.15975(20.163}$$

$$\begin{array}{r}
 2159 \\
 8347 \\
 3975 \\
 \hline
 0000
 \end{array}$$

In this Example, 3, the Unit's Place of the Divisor, falls under 6, the Ten's Place of the Dividend; therefore (by the first Rule) the first Figure in the Quotient is Tens: Or, by the second Rule, the Excess of Decimal Places in the Dividend, above the Divisor, is three; there being five Places of Decimals in the Dividend, and but two in the Divisor, so there must be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

$$375.89 \overline{) 15.675159(.0417}$$

$$\begin{array}{r}
 63955 \\
 263669 \\
 \hline
 546
 \end{array}$$

In this Example, 5, in the Unit's Place of the Divisor, falls under 7, the second Place of Decimals in the Dividend; therefore (by the first Rule) the first Figure in the Quotient is in the second Place of Decimals; so that you must put a Cypher before the first Figure in the Quotient; and by the second Rule, the Excess of decimal Places in the Dividend above the Number of decimal Places in the Divisor is 4; for the decimal Places in the Dividend is 6, and the Number of Places in the Divisor but two; therefore there must be four Places of Decimals in the Quotient: But the Division being finished after the common Way, the Figures in the Quotient are but three; therefore you must prefix a Cypher before the significant Figures.

Example 5. Let 72.1564 be divided by .1347.

$$.1347)72.1564(535.68$$

$$\begin{array}{r} \dots \\ \hline 4806 \\ 7654 \\ 9190 \\ 11080 \\ \hline 304 \end{array}$$

In this Example, the Divisor being a Decimal, the first Figure thereof falls under the Ten's Place in the Dividend, therefore the Units (if there had been any) should fall under the Hundred's Place in the Dividend, and so the first Figure in the Quotient is Hundreds. And, by the second Rule, there being four Places of Decimals in the Dividend, and as many in the Divisor, so the Excess is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Division to two Places further; so I have two Places of Decimals. See the Work.

Example

Example 6. Let .125 be divided by .0457.

.0457).1250000(2.735

....

914

3360

3199

1610

1371

3390

2285

105

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to *Contracted Division*.

.00456).0000059791(.00131

1419

511

55

Let

Let it be divided by 282.

$$282 \overline{) 1.0000000} (0.035461 \text{ series}$$

.....

1540

1300

1720

280

$$325 \overline{) .400000} (1.2307$$

750

1000

2500

225

$$42 \overline{) 495.00000} (11.785.71$$

.....

75

330

360

240

300

60

18

Division of DECIMALS contracted.

IN Division of Decimals the common Way, when the Divisor hath many Figures, and it is required to continue the Division till the Value of the Remainder be but small, the Operation will sometimes be large and tedious, but may be excellently contracted by the following Method.

The R U L E.

By the first Rule of this Chapter (Page 20), find what is the Value of the first Figure in the Quotient; then, by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the Left-hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Dividend as will answer them; and, in dividing, omit one Figure of the Divisor at each following Operation. A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.257432; and let there be three Places of Decimals in the Quotient.

2.257432

2.25743)721.175(62)319.467

..... 677229

43946

22574

21372

20317

1055

903

152

135

17

15

2

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend, and it is required, that three Places of Decimals be in the Quotient, so there must be six Places in all; that is, three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how often the Divisor is in the Dividend, and the Answer is three times; put 3 in the Quotient, and multiply and subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how often the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and subtract, and the next Remainder is 21372. Then prick off the 4 in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient; multiply thus, saying 9

D 2

times

times 4 is 36, for which I carry 4 (in respect of the 4 last pricked off), and 9 times 7 is 63, and 4 is 67; set down 7, and carry 6; and so proceed till the Division be finished, always respecting the Increase made from the Figures pricked off. Observe the Work, which will better inform you than many Words.

$$2.25743 \overline{) 721.17562 (319.467}$$

$$\begin{array}{r}
 677229 \\
 \hline
 439466 \\
 225743 \\
 \hline
 2137232 \\
 2031687 \\
 \hline
 1055450 \\
 902972 \\
 \hline
 1524780 \\
 1354458 \\
 \hline
 1703220 \\
 1580201 \\
 \hline
 123019
 \end{array}$$

I have set down the Work of this last Example at large, according to the common Way, that thereby the Learner may see the Reason of the Rule, all the Figures on the Right Side the perpendicular Line being wholly omitted.

Example.

Example 2. Let 5171.59165 be divided by 8.758615; and let it be required, that four Places of Decimals be pricked off in the Quotient.

$$8.758615)5171.5916\dot{5}(590.4577$$

.....

$$\underline{43793075}$$

$$7922841$$

$$\underline{7882754}$$

$$40087$$

$$\underline{35034}$$

$$5053$$

$$\underline{4379}$$

$$674$$

$$\underline{613}$$

$$61$$

$$\underline{61}$$

..

In this Example, I can't have 8, the first Figure in the Divisor, in 5, the first Figure of the Dividend; so that the Unit's Place of the Divisor falls under the Hundred's Place of the Dividend; so that there will be seven Figures in the Quotient; that is, three of whole Numbers, and four of Decimals; therefore there must be seven Figures in the Divisor (because the Number of Places in the Divisor and Quotient will be equal), and there must be eight Places in the Dividend; so that I cut off the Figure 5 with a Dash, as useless. Thus having proportioned the Dividend to the Divisor, and both to the Number of Places or Figures desired in the Quotient, I proceed to divide as before; saying how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and subtract,

D 3

tract, and the Remainder is 7922841. Then I prick off the first Figure in the Divisor, 5, and seek how often the remaining Figures of the Divisor in the aforesaid Remainder, which I find nine times; put 9 in the Quotient, and multiply thereby, saying 9 times 5 (the Figure pricked off) is 45, for which I carry 5, and say 9 times 1 is 9, and 5 I carry is 14; set down 4, and carry 1, and proceed to multiply the rest of the Figures, and subtract, and the Remainder will be 40087. Then prick off the Figure 1, and seek how often 87586 in the Remainder 40087, the Answer will be 0; so put 0 in the Quotient, and prick off the Figure 6, and seek how often 8758 in 40087, which will be four times; put 4 in the Quotient, and multiply, saying, 4 times 6 (the Figure last pricked off) is 24, for which I carry 2, and say 4 times 8 is 32, and 2 I carry is 34; set down 4, and carry 3; multiply the rest of the Figures, and subtract as before, and so proceed after the same manner, until all the Figures of the Divisor be pricked off, to the last Figure. See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example, 7, the Unit's Place of the Divisor, falls under 1, the first Place of Decimals; therefore the first Figure of the Quotient is in the first Place of Decimals; so the Quotient will be all Decimals. Then, because the Quotient Figures, and the Figures of the Divisor will be of an equal Number, dash off the 43 in the Divisor, and the 7 in the Dividend, as useless, and divide as before.

tract and the Remainder is 25. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 9 times, and carry 9 times to the Quotient, and subtract 81 from the said Remainder, and the Remainder is 4. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 5 times, and carry 5 times to the Quotient, and subtract 45 from the said Remainder, and the Remainder is 13. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 1 time, and carry 1 time to the Quotient, and subtract 1 from the said Remainder, and the Remainder is 6. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 7 times, and carry 7 times to the Quotient, and subtract 49 from the said Remainder, and the Remainder is 11. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 6 times, and carry 6 times to the Quotient, and subtract 36 from the said Remainder, and the Remainder is 5. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 8 times, and carry 8 times to the Quotient, and subtract 64 from the said Remainder, and the Remainder is 1. Then prick off the first Figure in the Divisor, and ask how often the remaining Part of the Dividend is contained in the said Remainder, saying 1 time, and carry 1 time to the Quotient, and subtract 1 from the said Remainder, and the Remainder is 0. The Answer will be 25.136176.

Altho' I have hitherto given Directions for proportioning the Divisor and Dividend, so as to bring into the Quotient what Number of Decimals you please, yet there is no absolute Necessity for it; but you may carry on your Division to what Degree you please, before you begin to prick off the Figures of the Divisor, in order to contract the Work, as in the following Examples, where it is not required to prick off any determinate Number of Decimals, but it may be done according to Discretion.

2.756756)7414.76717(2689.67118

.....

5513512

19012551

16540536

24720157

22054048

2666109

2481080

185029

165405

19624

19297

327

276

51

28

23

22

1

12.34254

12.34254)514.75498(41.705757
... 4937016

2105338
1234245

871084
863978

7106
6171

935
864

71
62

9
8

1



C H A P. VII.

Extraction of the SQUARE ROOT.

If a Square Number be given ;

TO find the Root thereof, that is, to find out such a Number, as being multiplied into itself, the Product shall be equal to the Number given ; such Operation is called, *The Extraction of the Square Root* ; which to do, observe the following Directions.

1st.

34 *Extraction of the Square Root. Part I.*

1st, You must point your given Number; that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and so upon every second Figure throughout.

2^{dly}, Then seek the greatest square Number in the first Point towards the Left-hand, placing the square Number under the first Point, and the Root thereof in the Quotient, and subtract the said square Number from the first Point, and to the Remainder bring down the next Point, and call that the Resolvend.

3^{dly}, Then double the Quotient, and place it, for a Divisor, on the Left-hand of the Resolvend; and seek how often the Divisor is contained in the Resolvend (reserving always the Unit's Place), and put the Answer in the Quotient, and also on the Right-hand Side of the Divisor; then multiply by the Figure last put in the Quotient, and subtract the Product from the Resolvend (as in common Division), and bring down the next Point to the Remainder (if there be any more), and proceed as before.

A TABLE of SQUARES and CUBES, and their ROOTS.

| Root | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|---|---|----|----|-----|-----|-----|-----|-----|
| Square | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |
| Cube | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 521 | 729 |

Example 1. Let 4489 be a Number given, and let the square Root thereof be required.

$$\begin{array}{r}
 4489\ 67 \\
 \underline{36} \\
 127.889 \text{ Resolvend.} \\
 \underline{889 \text{ Product.}}
 \end{array}$$

...

First,

Chap. 7. *Extraction of the Square Root.* 35

First, Point the given Number, as before directed; then, by the little Table foregoing, seek the greatest square Number in 44 (the first Point to the Left-hand), which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the Right-hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12; which place on the Left-hand for a Divisor, and seek how often 12 in 88 (reserving the Unit's Place), the Answer is 7 times; which put in the Quotient, and also on the Right-hand Side of the Divisor, and multiply 127 by 7, as in common Division, and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finished; and the square Root of 4489 is 67; which Root if you multiply by itself, that is 67 by 67, the Product will be 4489, equal to the given square Number, and prove the Work to be right.

Example 2. Let 106929 be a Number given, and let the square Root thereof be required.

106929(.327

9

62)169 Resolvend.

124 Product.

647)4529 Resolvend.

4529 Product.

....

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10 (the first Point), which by the little

36 *Extraction of the Square Root. Part I.*

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then subtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Resolvend; then double the Quotient 3, and it makes 6, which place on the Left-hand of the Resolvend for a Divisor, and seek how often 6 in 16; the Answer is twice; put 2 in the Quotient, and also on the Right-hand of the Divisor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which subtract from the Resolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Resolvend. Then double the Quotient 32, and it makes 64, which place on the Left Side of the Resolvend for the Divisor, and seek how often 64 in 452, which you'll find 7 times; put 7 in the Quotient, and also on the Right-hand of the Divisor, making it 647, which multiplied by the 7 in the Quotient, it makes 4529, which subtracted from the Resolvend, there remains nothing. So 327 is the Square Root of the given Number.

Example 3. Let 2268741 be a square Number given, the Root whereof is required.

$$\begin{array}{r}
 \begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \\
 2268741 \cdot 1506.23 \\
 \hline
 1
 \end{array} \\
 \begin{array}{r}
 25 \overline{) 126} \\
 \underline{125} \\
 1
 \end{array} \\
 \begin{array}{r}
 3006 \overline{) 18741} \\
 \underline{18036} \\
 705
 \end{array} \\
 \begin{array}{r}
 30122 \overline{) 70500} \\
 \underline{60244} \\
 10256
 \end{array} \\
 \begin{array}{r}
 301243 \overline{) 1025600} \\
 \underline{903729} \\
 121871
 \end{array} \\
 \text{Remains } .121871
 \end{array}$$

Having

Chap. 7. *Extraction of the Square Root.* 37

Having pointed the given Number as before directed, seek what is the greatest square Number in the first Point 2, which is 1; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; subtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and set on the Right-hand, making it 126; double the 1 in the Quotient; which makes 2; set 2 on the Left-hand for a Divisor, and ask how often 2 in 12, which will be 5 times; put 5 in the Quotient, and also on the Right-hand of the Divisor, making it 25; multiply (as in common Division) 25 by 5, and subtract the Product 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Resolvend; and double the 15 in the Quotient, it makes 30 for a new Divisor. Then seek how often 30 in 18, which you can't have; so that you must put 0 in the Quotient, and also on the Right-hand of the Divisor, and bring down the next Point, and it makes 18741 for another new Resolvend. Then seek how often 300 in 1874, which will be six times; put 6 in the Quotient, and also on the Right-hand of the Divisor; multiply and subtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a time, to the Remainders, and so prosecute the Work to what Number of decimal Parts you please; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Divisor: Then seek how often 3012 in 7050 (rejecting the Unit's Place), which will be twice; put 2 in the Quotient, and also on the Right hand of the Divisor, and multiply and subtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the square Root of the given Number is 1506.23, which being squared, or multiplied by itself, and the last Remainder added, will make the given Number as follows:

38 *Extraction of the Square Root. Part I.*

$$\begin{array}{r}
 1506.23 \\
 1506.23 \\
 \hline
 451869 \\
 301246 \\
 903738 \\
 7531150 \\
 150623 \\
 \hline
 2268728.8129 \\
 \text{The Remainder add } 12.1871 \\
 \hline
 \text{Proof } 2268741.0000
 \end{array}$$

Some more Examples for Practice.

Example 1. $\cdot \cdot \cdot \cdot$
 7596796(2756.228 Root.

$$\begin{array}{r}
 4 \\
 \hline
 47)359 \\
 329 \\
 \hline
 545)3067 \\
 2725 \\
 \hline
 5506)34296 \\
 33036 \\
 \hline
 51122)126000 \\
 110244 \\
 \hline
 551242)1575600 \\
 1102484 \\
 \hline
 5512448)47311600 \\
 44099584 \\
 \hline
 3212016
 \end{array}$$

Example

Example 2. $\overset{\cdot}{7}\overset{\cdot}{5}\overset{\cdot}{1}\overset{\cdot}{4}\overset{\cdot}{1}\overset{\cdot}{7}\overset{\cdot}{.}\overset{\cdot}{5}\overset{\cdot}{7}\overset{\cdot}{4}\overset{\cdot}{5}$ (866.4 Root.
64

$$\begin{array}{r}
 \hline
 166)1114 \\
 \underline{996} \\
 1726)11817 \\
 \underline{10356} \\
 17328)146157 \\
 \underline{138624} \\
 173364)753345 \\
 \underline{693456} \\
 59889 \\
 \hline
 \end{array}$$

If the given Number be a mixed Number, *viz.* consisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, &c. that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

40 *Extraction of the Square Root. Part I.*

Example 3. Let 656714.37512 be given, to find the square Root.

$$\begin{array}{r}
 \begin{array}{cccccccc}
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 6 & 5 & 6 & 7 & 1 & 4 & . & 3 & 7 & 5 & 1 & 2 & 0
 \end{array}
 \begin{array}{l}
 (810.379 \text{ Root.} \\
 64 \\
 \hline
 161 \overline{) 167} \\
 161 \\
 \hline
 16203 \overline{) 61437} \\
 48609 \\
 \hline
 162067 \overline{) 1282851} \\
 1134469 \\
 \hline
 1620749 \overline{) 14838220} \\
 14586741 \\
 \hline
 \text{Remains } 251479
 \end{array}
 \end{array}$$

In this Example there are five Places of Decimals; therefore put a Cypher to it, to make it even, that so there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a Decimal Fraction, the Work differs nothing from the Examples aforegoing, only you must be mindful to point your given Number right; for (as was before directed) the Number of Places must always be made even, and then begin to point at the Right-hand, as in whole Numbers.

If it be a Vulgar Fraction, it must be reduced to a Decimal, by the first Rule of the second Chapter.

I shall give an Example or two in each Case, and so conclude this Chapter.

Let

Chap. 7. *Extraction of the Square Root.* 41

Let .125 be a Decimal Fraction given, whose square Root is required; and let it be required to have four Places of Decimals in the Root.

$$\begin{array}{r}
 .12500000(.3535 \\
 \underline{9} \\
 65)350 \\
 \underline{325} \\
 703)2500 \\
 \underline{2109} \\
 7065)39100 \\
 \underline{35325} \\
 3775
 \end{array}$$

In this Example there must be five Cyphers annexed, because two Places in the Square make but one in the Root.

Let the square Root of .00715 be required.

$$\begin{array}{r}
 .007150(.084 \\
 \underline{64} \\
 164)750 \\
 \underline{656} \\
 94
 \end{array}$$

In this a Cypher is added to make the Place even.

42 *Extraction of the Square Root. Part I.*

Let $\frac{7}{8}$ be a Vulgar Fraction given, whose square Root is required.

| | |
|----------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 8 \overline{) 7000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ .. \end{array}$ | $\begin{array}{r} .87500000(.9354 \\ \underline{81} \\ 183 \overline{) 650} \\ \underline{549} \\ 1865 \overline{) 10100} \\ \underline{9325} \\ 18704 \overline{) 77500} \\ \underline{74816} \\ 2684 \end{array}$ |
|----------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875; to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{100}$ be a Vulgar Fraction, whose square Root is required.

| | |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 96 \overline{) 3.000000} \\ \underline{288} \\ 120 \\ \underline{96} \\ 240 \\ \underline{192} \\ 480 \\ \underline{480} \\ ... \end{array}$ | $\begin{array}{r} .00312500(.0559 \text{ Root.} \\ \underline{25} \\ 105 \overline{) 625} \\ \underline{525} \\ 1109 \overline{) 10000} \\ \underline{9981} \\ 19 \end{array}$ |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

In

Chap. 8. *Extraction of the Cube Root.* 43

In extracting the Root of this, because the first Point consists of Cyphers, there must be a Cypher put first in the Quotient.

To prove this Rule, square the Root, and to the Product add the Remainder, as was before directed. To square a Number, is to multiply it by itself; and to cube it, is to multiply the Square of the Number by the Number itself.



C H A P. VIII.

Extraction of the CUBE ROOT.

TO extract the Cube Root, is nothing else but to find such a Number, as being first multiplied into itself, and then into that Product, produceth the given Number; which to perform, observe the following Directions.

1st, You must point your given Number, beginning with the Unit's Place, and make a Point, or Dot, over every third Figure towards the Left-hand.

2^{dly}, Seek the greatest Cube Number in the first Point, towards the Left-hand, putting the Root thereof in the Quotient, and the said Cube Number under the first Point, and subtract it therefrom, and to the Remainder bring down the next Point, and call that the Resolvend.

3^{dly}, Triple the Quotient, and place it under the Resolvend; the Unit's Place of this under the Ten's Place of the Resolvend; and call this the Triple Quotient.

4^{thly}, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of
this

44 *Extraction of the Cube Root. Part I.*

this under the Ten's Place of the triple Quotient, and call this the Triple Square.

5thly, Add these two together, in the same Order as they stand, and the Sum shall be the Divisor.

6thly, Seek how often the Divisor is contained in the Resolvend, rejecting the Unit's Place of the Resolvend (as in the Square Root), and put the Answer in the Quotient.

7thly, Cube the Figure last put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Resolvend.

8thly, Multiply the Square of the Figure last put in the Quotient, into the triple Quotient, and place the Product under the last, one Place more to the Left-hand.

9thly, Multiply the triple Square by the Figure last put in the Quotient, and place it under the last, one Place more to the Left-hand.

10thly, Add the three last Numbers together, in the same Order as they stand, and call that the Subtrahend.

Lastly, Subtract the Subtrahend from the Resolvend, and if there be another Point, bring it down in the Remainder, and call that a new Resolvend, and proceed in all Respects as before.

Example

Chap. 8. *Extraction of the Cube Root.* 45

Example 1. Let 314432 be a Cubic Number, whose Root is required.

314432 (68 Root.

216

98432 Resolvend.

18 Triple Quotient of 6.

108 Triple Square of the Quotient 6:

1098 Divisor.

512 Cube of 8, the last Figure of the Root.

1152 The Square of 8, by the triple Quotient.

864 The triple Square of the Quotient 6 by 8.

98432 The Subtrahend.

.....

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table, (*Page 34*), you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolvend. Then triple the Quotient 6, it makes 18, which write down the Unit's Place, 8, under 3, the Ten's Place of the Resolvend. Then square the Quotient 6, and triple the Square, and it makes 108, which write under the triple Quotient, one Place toward the Left-hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contained in the Resolvend, (rejecting the Unit's Place thereof), that is, how often 1098 in 9843, which is 8 times;

46 *Extraction of the Cube Root. Part I.*

8 times; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18; the Product is 1152; set this under the Cube of 8, the Units of this under the Tens of that. Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864; set this down under the last Product, a Place more to the Left-hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is called the Subtrahend; which being subtracted from the Resolvend, the Remainder is nothing; which shews the Number to be a true cubic Number, whose Root is 68; that is, if 68 be cubed, it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624; and this Product, multiplied again by 68, the last Product is 314432, which shews the Work to be right.

| | |
|-----------|--------|
| | 68 |
| | 68 |
| | <hr/> |
| | 544 |
| The Work | 408 |
| | <hr/> |
| | 4624 |
| | 68 |
| | <hr/> |
| | 36992 |
| | 27744 |
| | <hr/> |
| The Proof | 314432 |

Example 2. Let the Cube Root of 5735339 be required.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which (by the little Table, *Page 34*) you will find to be 1; which place under 5, and 1, the Root thereof,
in

Chap. 8. *Extraction of the Cube Root.* 47

in the Quotient; and subtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Resolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which set one under another, in their Order, and added, makes 33 for the Divisor. Seek how often the Divisor in the Resolvend, and proceed as in the last Example.

5735339 (179 Root.

4735 Resolvend.

3 The Triple of the Quotient 1, the first Figure.

3 The triple Square of the Quotient 1.

33 The Divisor.

343 The Cube of 7, the second Figure of the Root.

147 The Square of 7, multipl. in the triple Quot. 3.

21 The triple Square of the Quot. multiplied by 7.

3913 The Subtrahend.

822339 The new Resolvend.

51 The Triple of the Quot. 17, the two first Fig.

867 The triple Square of the Quotient 17.

8721 Divisor.

729 The Cube of 9, the last Figure of the Root.

4131 The Squ. of 9, multipl. by the triple Quot. 51.

7803 The triple Square of the Quotient 867 by 9.

822339 The Subtrahend.

In

48 *Extraction of the Cube Root. Part I.*

In this Example, 33, the first Divisor, seems to be contained more than seven times in 4735, the Resolvend; but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Resolvend.

Some more Examples for Practice.

32461759(319 Root:

27

5461 Resolvend.

9 The Triple of 3.

27 The triple Square of 3.

279 The Divisor.

1 The Cube of 1, the second Figure.

9 The triple Quotient, by the Square of 1.

27 The triple Square, multiplied by 1, the 2d Fig.

2791 The Subtrahend.

2670759 A new Resolvend.

93 The Triple of 31.

2883 The triple Square of 31:

28923 The Divisor.

729 The Cube of 9, the last Figure.

7533 The Square of 9, by 93 the triple Quotient.

25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

.....

Chap. 8. *Extraction of the Cube Root.* 49

84604519(439 Root.

64

20604 Resolvend.

12 Triple of 4.
48 Triple Square of 4.

492 Divisor.

27 Cube of 3.
108 Square of 3, by the triple Quotient:
144 Triple Square by 3.

15507 Subtrahend.

5097519 Resolvend.

129 Triple of 43.
5547 Triple Square of 43.

55599 Divisor.

729 Cube of 9.
10449 Square of 9 by 129.
49923 Triple Square by 9.

4097519 Subtrahend.

.....

50 *Extraction of the Cube Root. Part I.*

259697989(638
216

43697 Resolvend.

18 Triple of 6.
108 Triple Square of 6.

1098 Divisor.

27 Cube of 3, the second Figure.
162 Triple Square of 3 by 18.
324 Triple Square 108 by 3.

34047 Subtrahend.

9650989 Resolvend.

189 Triple of 63.
11907 Triple Square of 63:

119259 Divisor.

512 Cube of 8.
12096 Square of 8 by 189.
95256 Triple Square 11907 by 8.

9647072 Subtrahend.

3917 Remainder.

Chap. 8. *Extraction of the Cube Root.* 51

25917056(295.9
8

17917 Resolvend.

6 Triple of 2.
12 Triple Square of 2.

126 Divisor.

729 Cube of 9, the 2d Figure.
486 Square of 9 by 6.
108 Triple Square by 9.

16389 Subtrahend.

1528056 Resolvend.
87 Triple of 29.
2523 Triple Square of 29.

25317 Divisor.

125 Cube of 5, the 3d Figure.
2175 Square of 5 by 87.
12615 Triple Square by 5.

1283375 Subtrahend.
244681000 Resolvend.

885 Triple of 295.
261075 Triple Square of 295.

2611635 Divisor.

729 Cube of 9, the last Figure.
71685 Square of 9 by 885.
2349675 Triple Square by 9.

235685079 Subtrahend.

8995921 Remainder.

F 2

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Resolvend; by which means I bring one Place of Decimals. And so you may proceed to more decimal Places at Pleasure, by annexing three Cyphers to the next Remainder, and carrying on the Work as before.

2206

52 *Extraction of the Cube Root.* Part I.

22069810125(2805
8

14069 Resolvend.

6 Triple of 2.

12 Triple Square of 2.

126 Divisor.

512 Cube of 8.

384 Square of 8 by 6.

96 Triple Square by 8.

13952 Subtrahend.

117810125 New Resolvend.

84 Triple of 28.

2352 Triple Square of 28.

23604 Divisor.

840 Triple of 280.

235200 Triple Square of 280.

2352840 New Divisor.

125 Cube of 5.

21000 Square of 5 by 840.

1176000 Triple Square by 5.

117810125 Subtrahend.

.....

In this Example
13952, being sub-
tracted from the
Resolvend 14069,
the Remainder is
117; to which
bring down 810,
the 3d Point, and
it makes 117810,
for a new Resol-
vend; and the next
Divisor is 23604,
which you cannot
have in the said
Resolvend (the
Unit's Place be-
ing rejected); so
you must put 0 in
the Quotient, and
seek a new Divisor
(after you have
brought down
your last Point to
the Resolvend);
which new Divisor
is 2352840; which
you will find to be
contained 5 times.
So proceed to fi-
nish the rest of the
Work.

93759-575070(45-42
64

29759 Resolvend.

12 Triple of 4, the first Figure.
48 Triple Square of 4.

492 Divisor.

125 Cube of 5, the 2d Figure.
300 Square of 5 by 12, the triple Quotient.
240 Triple Square by 5.

27125 Subtrahend.

2634575 Resolvend.

135 Triple of 45.
6075 Triple Square of 45.

60885 Divisor.

64 Cube of 4.
2160 Square of 4 by 135.
24300 Triple Square by 4.

2451664 Subtrahend.

182911070 Resolvend.

1362 Triple of 45-4.
618348 Triple Square of 45-4.

6184842 Divisor.

8 Cube of 2.
5448 Square of 2 by 1362.
236696 Triple Square of 2 by 2.

123724088 Subtrahend.

59186982 Remainder.

54 *Extraction of the Cube Root.* Part I.

In extracting the Cube Root of a mixed Number, always observe to make the decimal Part consist of either three, six, nine, &c. Places, that is, always to consist of even Points, as in the last Example, where the decimal Places were five; to which I annexed a Cypher to make up six, and so I proceed to point it; and by that means I have a Point fall upon the Unit's Place of whole Numbers, which you must always observe.

To extract the Cube Root out of a Fraction.

This is the same to do as in whole Numbers, observe but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal must always consist of three, six, nine, &c. Places; and if it be not so, it must be made so, by annexing Cyphers, as is above said.

If the Cube Root of a Vulgar Fraction be required, you must first reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example

Chap. 8. *Extraction of the Cube Root.* 55

Example 1. Let the Cube Root of .401719179 be required.

.401719179(.737 Root.

343

58719 Resolvend.

21 Triple of 7.

147 Triple Square of 7:

1491 Divisor.

27 Cube of 3.

189 Square of 3 by 21.

441 Triple Square by 3.

46017 Subtrahend.

12702179 Resolvend.

219 Triple of 73.

15987 Triple Square of 73:

160089 Divisor.

343 Cube of 7.

10731 Square of 7 by 219.

111909 Triple Square by 7.

11298553 Subtrahend.

1406236 Remainder.

Example

56 *Extraction of the Cube Root. Part I.*

Example 2. Let the Cube Root of .0001416 be required.

.000141600(.052 Root.

125

16600 Resolvend.

15 Triple of 5.

75 Triple Square of 5.

765 Divisor.

8 Cube of 2.

60 Square of 2 by 15.

150 Triple Square by 2.

15608 Subtrahend.

992 Remainder.

Example 3. Let $\frac{2}{278}$ be a Vulgar Fraction, whose Cube Root is required.

By the first Rule of Chapter II. reduce the Vulgar Fraction to a Decimal.

276)5.000000000(.018115942

.....

2240

320

440

1640

2600

1160

560

8

.01811

.018115942(.262 Root.
8

10115 Resolvend.

6 Triple of 2.
12 Triple Square of 2.

126 Divisor.

216 Cube of 6.
216 Square of 6 by the Triple of 2.
72 Triple Square by 6.

9576 Subtrahend.

539942 Resolvend.

78 Triple of 26.
2028 Triple Square of 26.

20358 Divisor.

8 Cube of 2.
312 Square of 2 by 78.
4056 Triple Square 2028 by 2.

408728 Subtrahend.

131214 Remainder.

You may prove the Truth of the Work, by cubing the Root found, as was shewed in the first Example; and if any thing remains, add it to the said Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will

58 *Multiplication of Feet, &c. Part I.*

I will shew the Proof of the fifth Example (*Page 50*), the given Number being 259697989, whose Root is 638, it being a furd Number, there remains 3917.

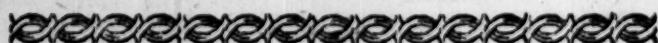
$$\begin{array}{r} 638 \\ 638 \\ \hline 5104 \\ 1914 \\ 3828 \\ \hline \end{array}$$

The Square 407044
638

$$\begin{array}{r} 3256352 \\ 1231132 \\ 2442264 \\ \hline \end{array}$$

The Cube 259694072
The Remainder add 3917

Proof equal to the given Numb. 259697989



C H A P. IX.

Multiplication of Feet, Inches, and Parts.

IN the multiplying of Feet, Inches, &c. I shall endeavour to lay down such easy and familiar Rules, as may easily be understood by the meanest Capacity.

Example

I. Chap. 9. *Multiplication of Feet, &c.* 59

Example 1. Let 7 Feet 9 Inches be multiplied by 3 Feet 6 Inches.

$$\begin{array}{r}
 \text{F. I.} \\
 7 \ 9 \\
 3 \ 6 \\
 \hline
 23 \ 3 \ \text{Pts.} \\
 3 \ 10 \ 6 \\
 \hline
 27 \ 1 \ 6
 \end{array}$$

First, Multiply 9 Inches by 3, saying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; set down 3 under Inches, and carry 2 to the Feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the Feet.

Then begin with 6 Inches, saying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; set down 6 Parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which set down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiplied by 9 Feet 8 Inches.

$$\begin{array}{r}
 \text{F. I.} \\
 75 \ 7 \\
 9 \ 8 \\
 \hline
 680 \ 3 \\
 50 \ 4 \ 8 \\
 \hline
 730 \ 7 \ 8
 \end{array}$$

First, Multiply by 9 Feet, saying, 9 times 7 is 63, which is 5 Feet 3 Inches; set down 3, and carry 5, saying, 9 times 5 is 45, and 5 I carry is 50; set down 0, and carry 5, saying, 9 times 7 is 63, and 5

is

60 *Multiplication of Feet, &c. Part II.*

is 68; set down 68, and proceed to multiply by 8 Inches, saying, 8 times 7 is 56; the Twelves in 56 are four times, and 8 remains; set 8 a Place to the Right-hand, and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient in 50 Feet, and 4 remains; set down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet 7 Inches 8 Parts.

I will repeat the last Example again, and shew another Way to work it, which, I think, is better, and more expeditious, when there are more Figures than one in the Feet; thus,

| | |
|-----|-----|
| F. | I. |
| 75 | 7 |
| 9 | 8 |
| | |
| 680 | 3 |
| 25 | 2 4 |
| 25 | 2 4 |
| | |
| 730 | 7 8 |

Multiply by 9 Feet, first, as above directed; then, instead of multiplying by 8 Inches, let the Inches be parted into such aliquot or even Parts of a Foot, as you find to be contained in that Figure; if you take such Parts of the Multiplicand, and add them to the former Product, the Sum will give the Answer: Thus, 8 Inches may be parted into 4, and 4, because 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and set it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the same as before; thus, say how often 3 in 7, which is twice; set down 2; then, because twice 3 is 6, say, 6 out of 7, and there remains 1, for which you must add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times; set down 5; and, because 3 times 5 is 15, there is 0 remains. Then go to the

Chap. 9. *Multiplication of Feet, &c.* 61

7 Inches, saying, the Threes in 7 are twice; set down 2 in the Inches; and because twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet 2 Inches 4 Parts; which set down again, and add all together, the Sum is 730 Feet 7 Inches 8 Parts; the same as before.

Example 3. Let 97 Feet 8 Inches be multiplied by 8 Feet 9 Inches.

| | |
|-------|----|
| F. | I. |
| 97 | 8 |
| 8 | 9 |
| <hr/> | |
| 781 | 4 |
| 48 | 10 |
| 24 | 5 |
| <hr/> | |
| 854 | 7 |

Begin, first, to multiply by 8 Feet, saying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; set down 4 Inches, and carry 5, saying, 8 times 7 is 56, and 5 I carry is 61; set down 1, and carry 6, saying, 8 times 9 is 72, and 6 I carry is 78, which set down: Then, instead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3; 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet 8 Inches, which is 48 Feet 10 Inches; and because 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work, as above.

62 Multiplication of Feet, &c. Part I.

Example 4. Let 75 Feet 9 Inches be multiplied by 17 Feet 7 Inches.

| F. | I. | |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|--|
| 75 | 9 | |
| 17 | 7 | |
| <div style="display: flex; justify-content: flex-end; align-items: center;"> <div style="margin-right: 20px;">525</div> <div style="margin-right: 20px;">75</div> <div style="margin-right: 20px;">25</div> <div style="margin-right: 20px;">3</div> <div>P.</div> </div> | | |
| <div style="display: flex; justify-content: flex-end; align-items: center;"> <div style="margin-right: 20px;">18</div> <div style="margin-right: 20px;">11</div> <div style="margin-right: 20px;">3</div> </div> | | |
| <div style="display: flex; justify-content: flex-end; align-items: center;"> <div style="margin-right: 20px;">8</div> <div style="margin-right: 20px;">6</div> </div> | | |
| <div style="display: flex; justify-content: flex-end; align-items: center;"> <div style="margin-right: 20px;">4</div> <div style="margin-right: 20px;">3</div> </div> | | |
| <div style="display: flex; justify-content: flex-end; align-items: center;"> <div style="margin-right: 20px;">1331</div> <div style="margin-right: 20px;">11</div> <div style="margin-right: 20px;">3</div> </div> | | |

In this Example, because there are more than 12 Feet in the Multiplier, therefore I first multiply the 75 by 17 Feet; then, because the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches (not meddling with the 7 Inches, because that was multiplied into the 9 before); then add all these together, and the Sum is 1331 Feet 11 Inches 3 Parts.

Example

I. by Chap. 9. *Multiplication of Feet, &c.* 63

Example 5. Let 87 Feet 5 Inches be multiplied by 35 Feet 8 Inches.

| F. | I. | |
|-------|----|----|
| 87 | 5 | |
| 35 | 8 | |
| <hr/> | | |
| 435 | | |
| 261 | | P. |
| 29 | 1 | 8 |
| 29 | 1 | 8 |
| 11 | 8 | 0 |
| 2 | 11 | 0 |
| <hr/> | | |

3117 10 4

Work here as in the last Example. After you have multiplied the Feet, then take the aliquot Parts of 8 Inches, which are two Thirds; therefore take the third Part of 87 Feet 5 Inches, and set it down twice. Thus the third Part of 87 Feet 5 Inches is 29 Feet 1 Inch 8 Parts; set this down twice; then the aliquot Parts of 5 Inches are 4 and 1, that is, a third Part and a 12th Part; therefore take a third Part of 35, which is 11 Feet 8 Inches, and a 12th Part of 35 is 2 Feet 11 Inches; set all these one under another, and add them together, and the Sum is 3117 Feet 10 Inches 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiplied by 48 Feet 11 Inches.

| F. | I. | |
|-------|----|----|
| 259 | 2 | |
| 48 | 11 | |
| <hr/> | | |
| 2072 | | |
| 1036 | | |
| 129 | 7 | P. |
| 86 | 4 | 8 |
| 21 | 7 | 2 |
| 8 | 0 | 0 |
| <hr/> | | |
| 12677 | 6 | 10 |
| G | 2 | |

First,

64 *Multiplication of Feet, &c.* Part I.

First, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, a half, a third, and a twelfth; therefore take the half of 297 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet 4 Inches 8 Parts, and the twelfth Part of 259 Feet 2 Inches is 21 Feet 7 Inches 2 Parts; or (because 1 is the fourth Part of 4), you may more readily take the fourth Part of 86 Feet 4 Inches 8 Parts, which is also 21 Feet 7 Inches 2 Parts; then 2 Inches are the sixth of 12, take the sixth of 48 Inches, which will be 8 Inches, which place under the Inches; then add all together, and the Sum is 12677 Feet 6 Inches 10 Parts. See the foregoing Work.

I shall set down only the working of some few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, &c.

| F. | I. |
|-----|----|
| 179 | 3 |
| 38 | 10 |

| | |
|------|-----|
| 1432 | |
| 537 | P. |
| 89 | 7 6 |
| 59 | 9 0 |
| 9 | 6 0 |

Product 6960 10 6

| F. | I. |
|-----|----|
| 246 | 7 |
| 36 | 9 |

| | |
|------|-----|
| 1476 | |
| 738 | P. |
| 123 | 3 6 |
| 61 | 7 9 |
| 12 | 0 0 |
| 9 | 0 0 |

Product 9061 11 3

| F. | I. |
|-----|----|
| 246 | 7 |
| 46 | 4 |

| | |
|------|-----|
| 1476 | |
| 984 | P. |
| 82 | 2 4 |
| 15 | 4 0 |
| 11 | 6 0 |

Product 11425 0 4

| F. | I. |
|-----|----|
| 257 | 9 |
| 39 | 11 |

| | |
|------|------|
| 2313 | |
| 771 | P. |
| 128 | 10 6 |
| 85 | 11 0 |
| 21 | 5 9 |
| 19 | 6 0 |
| 9 | 9 0 |

Product 10288 6 3

Example

Chap. 9. *Multiplication of Feet, &c.* 65

Example 11. Let 7 Feet 5 Inches 9 Parts be multiplied by 3 Feet 5 Inches 3 Parts.

| F. | I. | P. | | |
|-------|----|----|----|----|
| 7 | 5 | 9 | | |
| 3 | 5 | 3 | | |
| <hr/> | | | | |
| 22 | 5 | 3 | S. | |
| 3 | 1 | 4 | 9 | T. |
| | 1 | 10 | 5 | 3 |
| <hr/> | | | | |
| 25 | 8 | 6 | 2 | 3 |

In this Example, I first begin with 3 Feet, and thereby multiply 7 Feet 5 Inches 9 Parts: First, I say, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; set down 3 under the Parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; set down 5 Inches, and carry 1, and say, 3 times 7 is 21, and 1 I carry is 22; set down 22 Feet: Then begin with 5 Inches, saying 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; set down 9 Seconds a Place towards the Right-hand, and carry 3 Parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; set down 4 Parts, and carry 2, saying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; set down 3 Feet 1 Inch, and begin to multiply by 3 Parts, saying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; set down 3 Thirds, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; set down 5 Seconds, and carry 1, saying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which set down, and add all up, and the Product is 25 Feet 8 Inches 6 Parts 2 Seconds 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, &c. if Feet be multiplied by Feet, the Product is Feet; and Feet multiplied by Inches, the Product is Inches;

66 *Multiplication of Feet, &c.* Part I.

and the twelfth Part is Feet ; and Parts multiplied by Feet, the Product is Parts, and the twelfth Part thereof is Inches ; Parts multiplied by Inches, the Product is Seconds, and the twelfth Part thereof is Parts ; and Parts multiplied by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the first Row, and Parts by Inches in the second Row, and Parts by Parts in the third Row, the first Figure in every Row will stand a Place more towards the Right-hand, as you may see in the last Example.

Example 12. Let 37 Feet 7 Inches 5 Parts be multiplied by 4 Feet 8 Inches 6 Parts.

| F. | I. | P. | | |
|-------|----|----|----|----|
| 37 | 7 | 5 | | |
| 4 | 8 | 6 | | |
| <hr/> | | | | |
| 150 | 5 | 8 | S. | |
| 12 | 6 | 5 | 8 | |
| 12 | 6 | 5 | 8 | T. |
| 1 | 6 | 9 | 8 | 6 |
| <hr/> | | | | |
| 177 | 1 | 5 | 0 | 6 |

First, I multiply by 4 Feet, saying, 4 times 5 is 20, which is 1 Inch 8 Parts; set down 8, and carry 1, saying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet 5 Inches; set down 5 Inches, and carry 2, saying, 4 times 7 is 28, and 2 I carry is 30; set down 0, and carry 3, and say, 4 times 3 is 12, and 3 is 15; set down 15. Then I begin with 8 Inches; but, because the Feet in the Multiplicand are more than 12, it will be the best Way to work for the aliquot Parts of 8; so here I work for 4 Inches, and set that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet 7 Inches 5 Parts, which is twelve Feet six Inches five Parts eight Seconds; set this down twice: Then begin with 6 Parts; but, instead of multiplying, take half 37 Feet 7 Inches 5 Parts

Chap. 9. *Multiplication of Feet, &c.* 67

5 Parts (because 6 is half 12), and set it a Place more to the Right hand : Thus, the half of 37 Feet is 18, which I must count 18 Inches, because the Multiplier is 6 Parts ; so the half of 37 Feet 7 Inches 5 Parts, is one Foot six Inches nine Parts eight Seconds six Thirds ; which set down, and add all up together, and the Sum is 177 Feet 1 Inch 5 Parts 0 Seconds 6 Thirds.

Example 13. Let 311 Feet 4 Inches 7 Parts be multiplied by 36 Feet 7 Inches 5 Parts.

| F. | I. | P. | | | |
|-------|----|----|----|----|--|
| 311 | 4 | 7 | | | |
| 36 | 7 | 5 | | | |
| <hr/> | | | | | |
| 1866 | | | | | |
| 933 | | | | S. | |
| 103 | 9 | 6 | 4 | | |
| 77 | 10 | 1 | 9 | T. | |
| 8 | 7 | 9 | 6 | 4 | |
| 2 | 1 | 11 | 4 | 7 | |
| 12 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 0 | |
| | 9 | 0 | 0 | 0 | |
| <hr/> | | | | | |
| 11402 | 2 | 4 | 11 | 11 | |

In this Example, because the Feet both in the Multiplier and Multiplicand are compound Numbers, I first multiply the Feet one by the other ; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part ; so take the third Part of 311 Feet 4 Inches 7 Parts, which is 103 Feet 9 Inches 6 Parts 4 Seconds, and the fourth Part is 77 Feet 10 Inches 1 Part 9 Seconds ; set these down one under another, the Feet under the other Feet ; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part ; so the third Part of 311 Feet 4 Inches 7 Parts is 103 Feet 9 Inches 6 Parts 4 Seconds ; but, because the Multiplier is Parts, it must be set a Place

6

to

68 *Multiplication of Feet, &c. Part I.*

to the Right hand, that is, the 103 must be Inches, which is 8 Feet 7 Inches; therefore I set down 8 Feet 7 Inches 9 Parts 6 Seconds 4 Thirds. Then, because 1 Inch is a fourth Part of 4 Inches, therefore I take a fourth Part of 8 Feet 7 Inches 9 Parts 6 Seconds 4 Thirds, which is 2 Feet 1 Inch 11 Parts 4 Seconds 7 Thirds, which is the same as if I had taken a twelfth Part of 311 Feet 4 Inches 7 Parts. Then for 4 Inches in the Multiplicand, instead of multiplying 36 Feet by it, take a third Part, because 4 Inches is the third Part of 12; so the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a third and a fourth; so the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all these together, and the Sum will be 11402 Feet 2 Inches 4 Parts 11 Seconds 11 Thirds.

Example 14. Let 8 Feet 4 Inches 3 Parts 5 Seconds 6 Thirds, be multiplied by 3 Feet 3 Inches 7 Parts 8 Seconds 2 Thirds.

| F. | I. | P. | S. | T. | | | | | |
|---------|----|----|----|----|---|---|----|----|---|
| 8 | 4 | 3 | 5 | 6 | | | | | |
| 3 | 3 | 7 | 8 | 2 | | | | | |
| <hr/> | | | | | | | | | |
| 25 | 0 | 10 | 4 | 6 | | | | | |
| 2 | 1 | 0 | 10 | 4 | 6 | | | | |
| | 1 | 10 | 6 | 0 | 2 | 6 | | | |
| | | 5 | 6 | 0 | 3 | 8 | 0 | | |
| | | | 1 | 4 | 8 | 6 | 11 | 0 | |
| <hr/> | | | | | | | | | |
| Product | 27 | 7 | 3 | 5 | 1 | 8 | 8 | 11 | 0 |

In this last Example there is no Difficulty, if you do but observe the former Directions, and set every Row a Place more to the Right-hand.

Chap. 9. *Multiplication of Feet, &c.* 69

I shall only set down the working of some few Examples more, and so conclude this Chapter.

| F. | I. | P. | |
|-------|----|----|------|
| 321 | 7 | 3 | |
| 9 | 3 | 6 | |
| <hr/> | | | |
| 2894 | 5 | 3 | S. |
| 80 | 4 | 9 | 9 T. |
| 13 | 4 | 9 | 7 6 |
| <hr/> | | | |
| 2988 | 2 | 10 | 4 6 |
| <hr/> | | | |

| F. | I. | P. | |
|-------|----|----|----|
| 42 | 7 | 8 | |
| 7 | 3 | 6 | |
| <hr/> | | | |
| 298 | 5 | 1 | |
| 10 | 7 | 11 | S. |
| 1 | 9 | 3 | 10 |
| <hr/> | | | |
| 310 | 10 | 10 | 10 |
| <hr/> | | | |

| F. | I. | P. | |
|-------|----|----|------|
| 124 | 7 | 9 | |
| 14 | 6 | 2 | |
| <hr/> | | | |
| 496 | | | |
| 124 | | | S. |
| 62 | 3 | 10 | 6 T. |
| 1 | 8 | 9 | 3 6 |
| 7 | 0 | 0 | 0 0 |
| 1 | 2 | 0 | 0 0 |
| | 7 | 0 | 0 0 |
| | 2 | 6 | 0 0 |
| <hr/> | | | |
| 1809 | 0 | 1 | 9 6 |
| <hr/> | | | |

| F. | I. | P. | |
|-------|----|----|------|
| 259 | 10 | 8 | |
| 18 | 5 | 4 | |
| <hr/> | | | |
| 2072 | | | |
| 259 | | | S. |
| 86 | 7 | 6 | 8 |
| 21 | 7 | 10 | 8 T. |
| 7 | 2 | 7 | 6 8 |
| 9 | 0 | 0 | 0 0 |
| 6 | 0 | 0 | 0 0 |
| 1 | 0 | 0 | 0 0 |
| <hr/> | | | |
| 4793 | 6 | 0 | 10 8 |
| <hr/> | | | |

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| F. | I. | P. | F. | I. | P. |
|-------|----|----|-------|----|----|
| 267 | 7 | 10 | 317 | 9 | 7 |
| 25 | 9 | 7 | 37 | 5 | 9 |
| <hr/> | | | <hr/> | | |
| 1335 | | | 2219 | | |
| 534 | | | 951 | | S. |
| 133 | 9 | 11 | 105 | 11 | 2 |
| 66 | 10 | 11 | 26 | 5 | 9 |
| 11 | 1 | 9 | 13 | 2 | 10 |
| 1 | 10 | 3 | 6 | 7 | 5 |
| 12 | 6 | 0 | 18 | 6 | 0 |
| 2 | 1 | 0 | 9 | 3 | 0 |
| 1 | 0 | 6 | 1 | 6 | 6 |
| | 8 | 1 | 3 | 1 | 0 |
| <hr/> | | | <hr/> | | |
| 6905 | 0 | 7 | 11910 | 9 | 11 |
| | 0 | 10 | | 1 | 3 |
| <hr/> | | | <hr/> | | |





THE
Complete Measurer, &c.

PART II.

CHAP. I.

Mensuration of SUPERFICIES.

Superficial Figures are all such as have only Length and Breadth, not having any commensurable Thickness.

§ I. Of a SQUARE.

* * * * * SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or square) Angles. To find the superficial Content thereof, this is

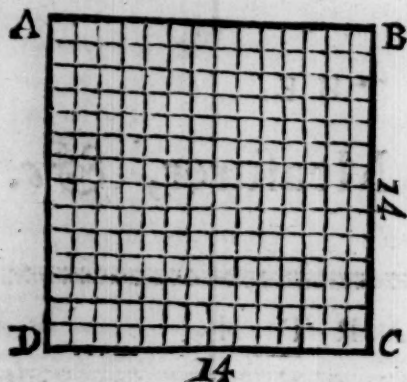
The RULE.

Multiply the Side into itself, and the Product is the Content.

Let

72 *Mensuration of Superficies.* Part II.

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Measure; multiply 14 by itself, and the Product is 196, which is the superficial Content.



$$\begin{array}{r}
 14 \\
 14 \\
 \hline
 56 \\
 14 \\
 \hline
 196 \text{ Product.}
 \end{array}$$

By Scale and Compasses.

Extend the Compasses from 1, in the Line of Numbers, to 14; the same Extent will reach from the same Point, turned forward to 196.

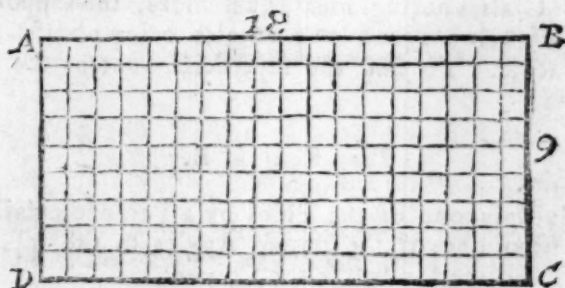
Demonstration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, crossing each other within the Square; so shall the whole great Square be divided into 196 little Squares, as you may see in the Figure above, equal to the Number of Square Feet, Yards, Poles, or other Measures, by which the Side was measured.

§ II. Of a PARALLELOGRAM, or
Long Square.

A Parallelogram is a Figure having four Sides, and as many Right Angles, the opposite Sides thereof being equal and parallel. To find the superficial Content thereof, this is

The R U L E.

Multiply the Length by the Breadth, and the Product is the superficial Content.



$$\begin{array}{rcl} \text{Length} & - & 18 \\ \text{Breadth} & - & 9 \\ \hline \text{Product} & - & 162 \end{array}$$

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet; which multiplied together, the Product is 162, the superficial Content thereof.

By Scale and Compasses.

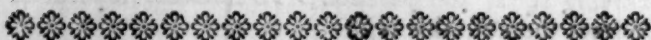
Extend the Compasses in the Line of Numbers from 1 to 9, the same Extent will reach from 18 down to 162, the square Feet.

H

Demon.

74 *Mensuration of Superficies. Part II.*

Demonstration. If the Sides AB and CD be each divided into 18 equal Parts, representing 18 Feet; and the Lines AD and BC each divided into 9 equal Parts, and Lines drawn from Point to Point, crossing each other within the Figure; those Lines will make thereby so many little Squares as there are square Feet, viz. 162.



§ III. *Of a R H O M B U S.*

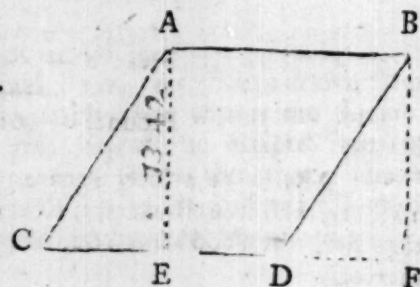
A Rhombus is a Figure representing a Quarry of Glas, having four equal Sides, the Opposites thereof being equal, two Angles being obtuse, and two acute. To find the superficial Content thereof, this is

The R U L E.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuse Angles to the opposite Side, and the Product is the Content.

| | |
|----------|-------|
| Perpend. | 13.42 |
| The Side | 15.5 |
| | <hr/> |
| | 6710 |
| | 6710 |
| | 1342 |
| | <hr/> |

Product 208.010



Let ABCD be a Rhombus given, whose Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiplied together, the Product is 208.010; which is the superficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By

By Scale and Compasses.

Extend the Compasses from 1 to 13.42, that Extent will reach from 15.5, the same Way to 208 Feet the Content.

Demonstration. Let CD be extended out to F, making DF equal to CE, and draw the Line BF; so shall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, because AB and CF are parallel. Therefore the Parallelogram ABFF is equal to the Rhombus ABCD.



§ IV. *Of a RHOMBOIDES.*

A Rhomboides is a Figure having four Sides, the opposite whereof are equal and parallel; and also four Angles, the opposite whereof are equal. To find the superficial Content thereof, this is

The RULE.

Multiply one of the longest Sides thereof by the Perpendicular let fall from one of the obtuse Angles to one of the longest Sides, and the Product is the Content.

$$\begin{array}{r}
 195 \\
 102 \\
 \hline
 390 \\
 195 \\
 \hline
 198.90 \\
 \text{H } 2
 \end{array}$$

Let



Let ABCD be a Rhomboides given, whose longest Sides, AB or CD, is 19.5 Feet, and the Perpendicular AE is 10.2; which multiplied together, the Product is 198.9, that is, 198 superficial Feet and 9 tenth Parts, the Content.

Demonstration. If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; so will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD; which was to be proved.



§ V. Of a TRIANGLE.

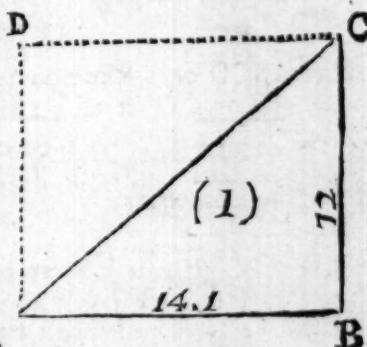
A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are such as have one Right Angle. Oblique-angled Triangles are such as have their Angles either acute or obtuse. An obtuse Angle is greater than a Right Angle, that is, it is more than 90 Degrees; and an acute Angle is less than a Right Angle. To find the superficial Content thereof, this is

The

The R U L E.

Let the Triangle be of what Kind soever, multiply the Base by half the Perpendicular, or half the Base by the whole Perpendicular; or, multiply the whole Base by the whole Perpendicular; and take half the Product; any of these three Ways will give the Content.

Let A B C be a Right-angled Triangle, whose Base is 14.1 Feet, and the Perpendicular 12 Feet. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or, multiply 14.1 by 12, the Product is 169.2; A the half thereof is 84.6, the same as before.

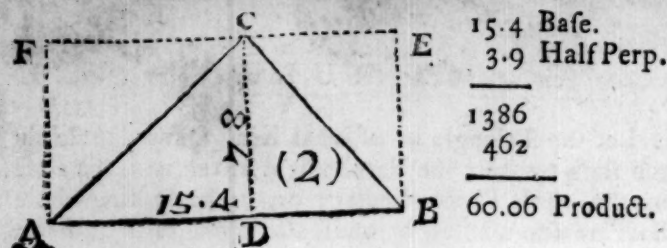


| |
|-----------------------|
| 14.1 Base. |
| 6 Half Perpendicular. |
| <hr/> |
| 84.6 Product. |

| |
|-------------------|
| 14.1 Base. |
| 12 Perpendicular. |
| <hr/> |
| 169.2 Product. |
| <hr/> |
| 84.6 Half. |

By Scale and Compasses.

Extend the Compasses from 2 to 14.1, that Extent will reach the same Way from 12 to 84.6 Feet, the Content.



| | |
|---------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| 15.4 Base. 7.8 Perpend. <hr/> 1232 1078 <hr/> 120.12 <hr/> 60.06 Half. | 7.7 Half Base. 7.8 Perpendicular. <hr/> 616 539 <hr/> 60.06 Product. |
|---------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|

Let ABC (*Fig. 2.*) be an oblique-angled Triangle given, whose Base is 15.4 , and the Perpendicular 7.8 ; if 15.4 be multiplied by 3.9 (half the Perpendicular), the Product will be 60.06 for the Area, or superficial Content: Or, if the Perpendicular 7.8 be multiplied into half the Base 7.7 , the Product will be 60.06 as before: Or, if 15.4 , the Base, be multiplied by the whole Perpendicular 7.8 , the Product will be 120.12 , which is the double Area; the Half thereof is 60.06 Feet, as before. See the Work.

By Scale and Compasses.

Extend the Compasses from 2 to 15.4 , that Extent will reach from 7.8 to 60 Feet, the Content.

Demonstration. If AD (*Fig. 1.*) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC shall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram is

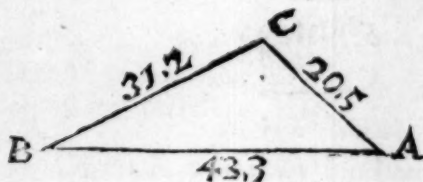
is the Area of the Triangle. In Fig. 2. the Parallelogram ABEF is also double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle: Which was to be proved.

To find the Area of any plain Triangle by having the three Sides given, without the Help of a Perpendicular.

The R U L E.

Add the three Sides together, and take half that Sum: Then subtract each Side severally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the last Product extract the Square Root; which Square Root shall be the Area of the Triangle sought.

Example. Let ABC be a Triangle, whose three Sides are as followeth; viz. AB 43.3, AC 20.5, and BC 31.2, the Area is required.



$$\text{Sides } \left\{ \begin{array}{l|l} 43.3 & 4.2 \\ 31.2 & 16.3 \\ 20.5 & 27.0 \end{array} \right\} \text{Differences.}$$

$$\text{Sum } \underline{95.0}$$

$$\text{Half } 47.5$$

Area

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Area 296.31

47.5 The half Sum.
27 Difference.

3325
950

1282.5 Product.
16.3 Difference.

38475
76950
12825

20904.75 Product.
4.2 Difference.

4180950
8361900

87799 9500(296.31
4

49)477
441

586)3699
3516

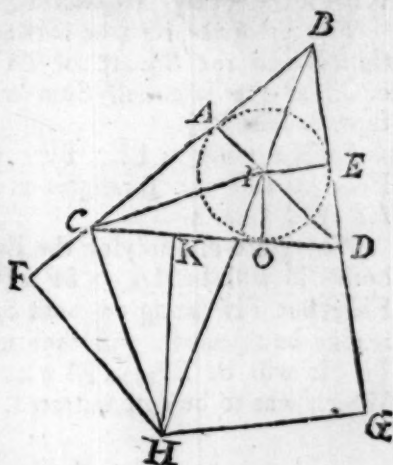
5923)18395
17769

59261)62600
59261

3339 Remains.

Demon-

Demonstration. In the Triangle BCD, I say, if from the half Sum of the Sides, you subtract each particular Side, and multiply the half Sum and the three Differences together continually, the Square Root of the Product shall be the Area of the Triangle.

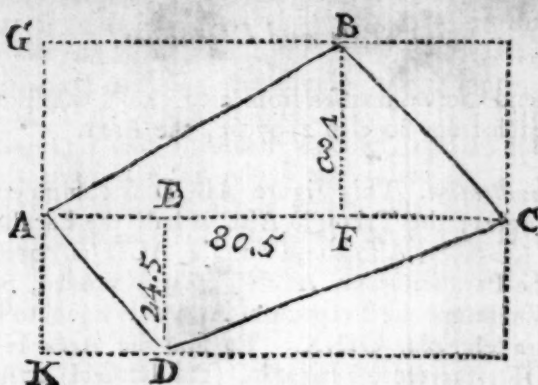


First, by the Lines BI, CI, and DI, bisect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI, and OI, being all equal to one another, because the Point I is the Center of the inscribed Circle (by *Euclid, Lib. IV. Prop. 4.*): Wherefore to the Side BC join CF equal to DE, or DO; so shall BF be equal to half the Sum of the Sides; *viz.* $=\frac{1}{2} BC + \frac{1}{2} BD + \frac{1}{2} CD$.

And $BA = BF - CD$; for $CA = CO$ and $OD = CF$; therefore $CD = AF$; and $AC = BF - BD$, for $BE = BA$, and $ED = CF$; therefore $BD = BA + CF$, and $CF = BF - BC$.

Then make $CK = CF$, and draw the Perpendiculars FH, GH, and KH, and extend BI to H; because the Angle $FCK + FHK$ are equal to two Right Angles (for the Angles F and K are Right Angles) equal also to $FCK + ACO$ by (*Euclid. I. 13.*), and the Angles $ACO + AIO$ are equal to two Right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are also similar. And the Triangles BAI and BFH are likewise similar.

From



$$BF = 30.1$$

$$DE = 24.5$$

$$\text{Sum } 54.6$$

$$\text{Half } 27.3$$

$$AC = 80.5$$

$$1365$$

$$1284$$

$$\text{Area } 2197.65$$

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; these two added together, the Sum is 54.6, the Half thereof is 27.3, which multiplied by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiplied by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the same as before.

By

By Scale and Compasses.

Extend the Compasses from 2 to 54.6; that Extent will reach from 80.5 to 2197.65, the Area.

Demonstration. This Figure ABCD is composed of two Angles; the Triangle ABC is half the Parallelogram AGHC: Also the Triangle ACD is equal to half the Parallelogram ACIK, as was proved, Sect. V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area $HI = BF + DE$; therefore $\frac{1}{2} HI \times AC (= KI = GH) = \text{Area of the Trapezium, which was to be proved.}$



§ VII. Of IRREGULAR FIGURES.

IRregular Figures are all such as have more Sides than four, and the Sides and Angles unequal. All such Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of such Figures, they must be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another; and so find the Areas of the Trapeziums and Triangles severally, and then add all the Areas together; so will you have the Area of the whole Figure.

Let ABCDEFG be an irregular Figure given to be measured; first, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Area severally.

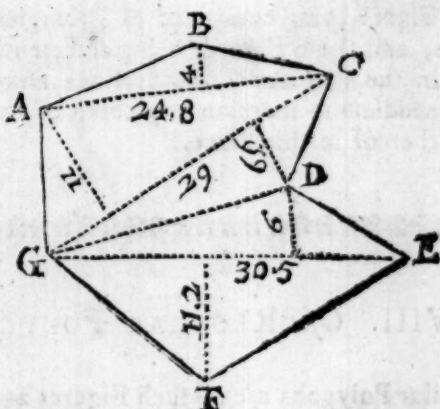
First, I multiply the Base AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then

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Then for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiplied by 29, the Diagonal; the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the half thereof is 8.6; which multiplied by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All these Areas added together, make 567.1, and so much is the Area of the whole irregular Figure. See the Work.



24.8 Base AC.
 2 Half Perpendicular.

49.6 Area of ABC.

11 Perpendicular.
 6.6

17.6 Sum.

8.8 Half.
 29 Diagonal CG.

792
 176

255.2 Area of ACGD.

I

11.2

| | |
|------------------------|-------------------------|
| 11.2 } Perpendiculars. | 30.5 |
| 6 } | 8.6 |
| <hr/> | |
| 17.2 Sum. | 1830 |
| | 2440 |
| <hr/> | |
| 8.6 Half Sum. | |
| | 262.30 Area of GDEF. |
| | 255.2 Area of ACGD. |
| | 49.6 Area of ABC. |
| <hr/> | |
| | 567.1 Sum of the Areas. |

This Figure being composed of Triangles and Trapeziums, and those Figures being sufficiently demonstrated in the Vth and VIth Sections aforegoing, it will be needless to mention any thing of the Demonstration thereof in this Place.



§ VIII. Of REGULAR POLYGONS.

Regular Polygons are all such Figures as have more than four Sides, all the Sides and Angles thereof being equal. Polygons are denominated from the Number of their Sides and Angles.

| | | | | | |
|---------------------------|---|----|------------------------------------------------|---|------------|
| If the Figure consists of | { | 5 | Equal Sides and Angles, it is called a Regular | { | Pentagon. |
| | | 6 | | | Hexagon. |
| | | 7 | | | Heptagon. |
| | | 8 | | | Octagon. |
| | | 9 | | | Enneagon. |
| | | 10 | | | Decagon. |
| | | 11 | | | Endecagon. |
| | | 12 | | | Dodecagon. |

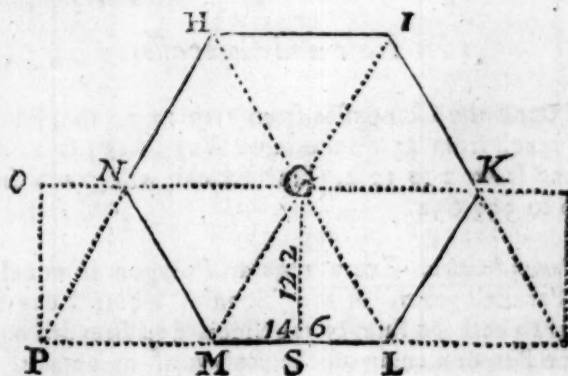
To

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To find the Area or superficial Content of any regular Polygon, this is

The R U L E.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular let fall from the Center to the Middle of one of the Sides ; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14.6

3

43.8 Half Sum of the Sides.

12.64 The Perpendicular.

43.8 Half Sum.

10112

3792

5056

553.632 Area.

14.6

6

87.6 Sum of the Sides.

6.32 Half Perpend.

1752

2628

5256

553.632 Area.

1.2

Let

Let HIKLMN be a Regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiplied by the Perpendicular GS 12.64, the Product is 553.632: Or, if 87.6, the whole Sum of the Sides, be multiplied by half the Perpendicular 6.32, the Product is 553.632, the same as before, which is the Area of the given Hexagon.

By Scale and Compasses.

Extend the Compasses from 1 to 12.2, that Extent will reach from 43.8, the same Way to 553.632: Or, extend from 2 to 12.2, that Extent will reach from 87.6 to 553.632.

Demonstration. Every regular Polygon is equal to the Parallelogram, or long Square, whose Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of six equilateral Triangles: And the Parallelogram OPQR is also composed of six equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

A TABLE for the more ready finding the Area of a Polygon.

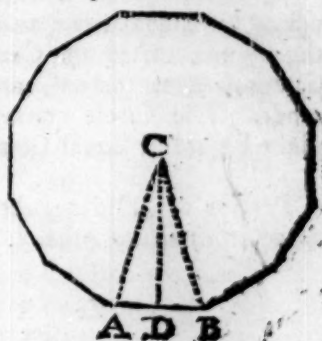
| Number of Sides. | Names. | Multipliers. |
|------------------|-----------|--------------|
| 3 | Trigon | .433013 |
| 4 | Tetragon | 1.000000 |
| 5 | Pentagon | 1.720477 |
| 6 | Hexagon | 2.598076 |
| 7 | Heptagon | 3.633959 |
| 8 | Octagon | 4.828427 |
| 9 | Enneagon | 6.181827 |
| 10 | Decagon | 7.694209 |
| 11 | Endecagon | 8.514250 |
| 12 | Dodecagon | 9.330125 |

Multiply the Square of the Side by the Tabular Number, and the Product is the Area of the Polygon.

How to find these Tabular Numbers.

These Numbers are found by Trigonometry, thus: Find the Angle at the Center of the Polygon by dividing 360 Degrees by the Number of Sides of the Polygon.

Example. Suppose each Side of the Dodecagon annexed be 1, and the Area be required,



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Divide 360 by 12 (the Number of Sides), and the Quotient is 30 Degrees for the Angle ACB; the Half thereof is 15, the Angle DCB, whose Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then say:

| | |
|------------------------------|-----------------|
| As s, DCB 15 Degrees is | Co-ar. 0.587004 |
| to .5 the Half-side DB. Log. | 1.698970 |
| so is s, CBD 75 Degrees, | 9.984944 |
| to the Perpendicular CD | 1.866025 |
| | <u>0.270918</u> |

Then 1.866025 multiplied by .5 (the Half-side) the Product is 9.330125 the Area of the Dodecagon required.



§ IX. *Of a CIRCLE.*

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure, called the Center, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compass.

Problem 1. Having the Diameter and Circumference, to find the Area.

The R U L E.

Every Circle is equal to a Parallelogram, whose Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply

ply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumf.
11.3 Half Diameter.

1065
355
355

401.15 Area.



Thus, if the Diameter of a Circle (that is, the Line drawn cross the Circle through the Center) be 22.6; and if the Circumference be 71, the Half of 71 is 35.5, and the Half of 22.6 is 11.3; which multiplied together, the Product is 401.15, which is the Area of the Circle.

Demonstration. Every Circle may be conceived to be a Polygon of an infinite Number of Sides, and the Semidiameter must be equal to the Perpendicular of such a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiplied by half the Diameter, gives the Area as aforesaid.

Or (with *F. Ignat. Gaston Pardies*) " Every Circle
" is equal to a Rectangle Triangle, one of whose
" Legs is the Radius, and the other a Right Line
" equal to the Circumference of the Circle: For
" such a Triangle will be greater than any Polygoa
" inscribed, and less than any Polygon circumscribed,
" by the 24th, 25th, 26th, and 27th Articles of the
" fourth Book of his Elements of Geometry; and
" therefore must be equal to the Circle.

" For

“ For (says he) should it be greater than the Circle,
 “ be the Excess as little as it will, a Polygon may be
 “ circumscribed, whose Difference, from the Circle,
 “ shall be yet less than the Difference between that
 “ Circle and the Rectangle Triangle; and that that
 “ Polygon will be less than the Triangle, is absurd;
 “ and if it be said, that this rectangled Triangle is
 “ less than the Circle, an inscribed Polygon may be
 “ made, which shall be greater than that Triangle;
 “ which is impossible.

“ This cannot but be admitted as a Principle, That
 “ if two determinate Quantities, A and B, are such
 “ that if every imaginable Quantity, which is greater
 “ or less than A, is also greater or less than B, these
 “ two Quantities A and B must be equal.

“ And this Principle being granted, which is in a
 “ manner self-evident, it may directly be proved, that
 “ the Triangle (before-mentioned) is equal to the
 “ Circle; because every imaginable inscribed Figure,
 “ which is less than the Circle, is also less than the
 “ Triangle; and every circumscribed Figure, greater
 “ than the Circle, is also greater than the Triangle.”

Problem 2. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, so is the Diameter to the Circumference.

Or, as 113 to 355, so is the Diameter to the Circumference.

Or, as 1 to 3.141593, so is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiplied by 22, and the Product is 497.2; which, divided by 7, gives 71.028 for the Circumference.

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ference. Or (by the second Proportion) if 22.6 be multiplied by 355, the Product will be 8023; this divided by 11.3, the Quotient is 71, the Circumference. Or (by the third Proportion) if 22.6 be multiplied into 3.141593, the Product is 71.0000018 the Circumference; which two last Proportions are the most exact.

| | |
|----------------|----------------|
| 22.6 | 355 |
| 22 | 22.6 |
| <hr/> | <hr/> |
| 452 | 2130 |
| 452 | 710 |
| <hr/> | <hr/> |
| 7)497.2(71.028 | 113)8023.01(71 |
| | 791 |
| | <hr/> |
| 3.141593 | 113 |
| 22.6 | 113 |
| <hr/> | <hr/> |
| 18849558 | ... |
| 6283186 | |
| 6283186 | |
| <hr/> | |
| 71.0000018 | |

By Scale and Compasses.

Extend the Compasses from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle to the Circumference was never yet exactly found, notwithstanding many eminent learned Men have laboured very far therein; among which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the said Proportion to 36 Places of Decimals, which are engraven upon his Tomb stone in *St. Peter's Church* in *Leyden*; which Numbers are these:

Diameter.

Diameter.

1.00000.00000.00000.00000.00000.00000.00000.

Circumference.

3.14159.26535.89793.23846.26433.83279.50288.

Of which large Number, these six Places, 3.14159, answering to the Diameter 1.00000, may be sufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I shall leave my Reader to use which of them he pleases, but shall commend the last two as most exact, tho' the first be most in Use: But in the following Work I shall use sometimes one of them, and sometimes another; but for the most part that of *Van Culen*, as being most exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309, so is the Circumference to the Diameter.

Or, as 355 to 113, so is the Circumference to the Diameter.

Or, as 22 to 7, so is the Circumference to the Diameter.

Let the Circumference be 71 (as in the former Circle), if .318309 be multiplied by 71 (as by the first Proportion), the Product will be 22.599939 for the Diameter. Or, by the second Proportion, 113 multiplied by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiplied by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309

| | | |
|-----------|---------------|--------------|
| .318309 | 113 | 71 |
| 71 | 71 | 7 |
| <hr/> | <hr/> | <hr/> |
| 318309 | 113 | 22)497(22.59 |
| 2228163 | 791 | 44 |
| <hr/> | <hr/> | <hr/> |
| 22.599939 | 355)8023(22 6 | 57 |
| | 710 | 44 |
| | <hr/> | <hr/> |
| | 923 | 130 |
| | 710 | 110 |
| | <hr/> | <hr/> |
| | 2130 | 200 |
| | 2130 | 198 |
| | <hr/> | <hr/> |
| | ... | 2 |

Thus, by both the first Proportions, the Diameter is 22.6, but by the last it falls something short.

By Scale and Compasses.

Extend the Compasses from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter sought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the same will reach from 71 to 22.6, as before.

Note, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters (by *Euclid*. 12.2). Now, the Area of a Circle, whose Diameter is 1, will be .785398, according to *Van Culen's* Proportion before-

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 before-mentioned; but for Practice .7854 will be sufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, so is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.15 (the Area of the given Circle): But,

According to *Metius's* Proportion,

As 452 : 355 :: 510.76 : 401.15, the same as before.

But, if you use *Archimedes's* Proportion, say,

As 14 : 11 :: 510.76 : 401.31; which Area is greater than by the two former Proportions; though in small Circles this is near enough the Truth. See the Working of all these.

22.6 Diameter of the former Circle.

22.6

1356

452

452

510.76 the Square of the said Diameter.

As 1 : .7854 :: 51076

.7854

204304

255380

408608

357532

401.150904 the Area.

By

By Scale and Compasses.

The Extent from 1 to 22.6, being twice turned over from .7854, will fall at the last upon 401.15, the Area.

$$\begin{array}{r}
 113 \\
 \hline
 4 \\
 \hline
 452
 \end{array}
 \qquad
 \begin{array}{r}
 \text{As } 452 : 355 :: 510.76 \\
 \hline
 355 \\
 \hline
 255380 \\
 255380 \\
 \hline
 153228 \\
 \hline
 452)181319.80(401.15 \\
 \underline{1808} \\
 519 \\
 \underline{452} \\
 678 \\
 \underline{452} \\
 2260 \\
 \underline{2260} \\
 \hline
 \dots
 \end{array}$$

As 14 : 11 :: 510.76

$$\begin{array}{r}
 11 \\
 \hline
 14 \overline{) 5618.36} (401.31 \\
 \underline{56} \\
 18 \\
 \underline{14} \\
 43 \\
 \underline{42} \\
 16 \\
 \underline{14} \\
 2
 \end{array}$$

Problem 5. Having the Circumference of a Circle to find the Area.

Because the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; so is the Diameter of another Circle to its Circumference: Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq. Cir. Area. Sq. Circumf.

As 1 : .07958 :: 5041

$$\begin{array}{r}
 5041 \\
 \hline
 7958 \\
 31832 \\
 397900 \\
 \hline
 401.16278
 \end{array}$$

22
4
88

4

Or

Or thus:

$$\text{As } 88 : 7 :: 5041$$

7

$$88)35287(400.98 \text{ Area.}$$

352

870

792

355

4

1420

780

704

76

Or, As 1420 : 113 :: 5041 : 401.15 Area.

Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

$$\text{As } 1 : .8862 :: 22.6 \text{ (the Diameter)}$$

22.6

53172

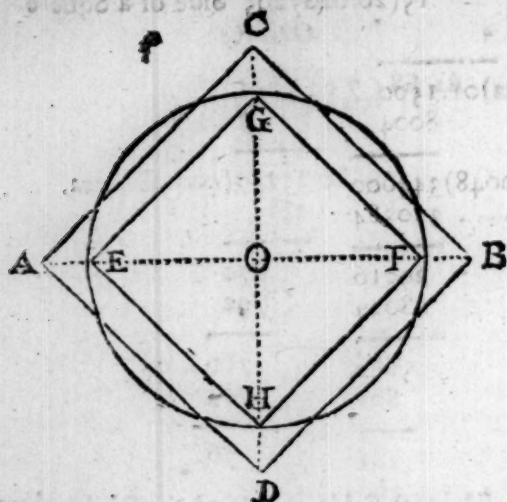
17724

17724

To 20.02812 the Side of the Square AC.

K 2

Let



Let the Diameter of the Circle EF or GH, be 22.6 (as before), to find the Side of the Square AC, AD, &c. If .8862 be multiplied by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiplied Square wise, that is, by itself, it will produce 401.1255907344, which is nearly equal to the Area found in the last Problem.

You may find the Side of the Square equal, by extracting the Square Root out of the Area of the given Circle.

401.15 (20.0287295 Side of a Square.

4

```

      4002)01.1500
            8004
            ---
      40048)349600
            .... 320384
            ---
              29216
              28034
              ---
                1182
                801
                ---
                 381
                 360
                 ---
                  21
                  20
                  ---
                   1
  
```

N. B. By this Method of extracting the Square Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

As 1 : .2821 :: 71 (the Circumference)

```

      71
      ---
    2821
    19747
    ---
  
```

20.0291 the Side of the Square.

K 3

Problem

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Problem 8. Having the Diameter, to find the Side of a Square, which may be inscribed in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inscribed will be .7071. Therefore,

As 1 : .7071 :: 22.6

22.6

$$\begin{array}{r} 42426 \\ 14142 \\ 14142 \\ \hline \end{array}$$

To 15.98046 the Side EG inscribed.

Or, if you square the Semidiameter and double that Square, the Square Root of the doubled Square will be the Side of the Square inscribed. For (by *Euclid*, 1.47), the Square of the Hypotenuse EG is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiameter.

11.3

$$\begin{array}{r} 339 \\ 113 \\ 113 \\ \hline \end{array}$$

127.69 the Square of EO, which double, be-
2 [cause EO=OG.

255.38 (15.98 Root, which is the Side of the Sq.

1

$$\begin{array}{r} 25)155 \\ 125 \\ \hline \end{array}$$

$$\begin{array}{r} 309)3038 \\ 2781 \\ \hline \end{array}$$

$$\begin{array}{r} 3188)25700 \\ 25504 \\ \hline \end{array}$$

196

Problem

Problem 9. Having the Circumference, to find the Side of a Square which may be inscribed.

If the Circumference be 1, the Side of the Square inscribed will be .2251. Therefore,

$$\text{As } 1 : .2251 :: 71$$

$$\begin{array}{r} 2251 \\ 15757 \end{array}$$

15.9821 the Side of the Sq. EG.

Because that in each of the four last Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be easy to work them with Scale and Compasses; for if you extend the Compasses from the first to the second, that Extent will reach from the third to the fourth: As in the last Problem, where the Proportion is as 1 to .2251, so is 71 to the Side of the Square 15.9821. Here extend the Compasses from 1 to .2251; that Extent will reach from 71 to 15.98; and so of the rest. But the fifth must be wrought like the fourth, thus; extend the Compasses from 1 to 71; that Extent, turned over the same Way from .07958, will fall, at last, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

Area.

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Area. Sq. Diam. Area.

As 1 : 1.2732 :: 401.15

401.15

63660

12732

12732

509280

510.744180 (22.599 the Diameter.

4

42)110

84

445)2674

2225

4509)44941

40581

45189)436080

406701

29379

By Scale and Compasses.

Extend the Compasses from 1 to 1.2732; that Extent will reach from 401.15 to 510.74, &c. Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and 7854, into two equal Parts, and one of those Parts will reach from 1 to 22.6, the Diameter sought.

Problem

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Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

Ar. Sq. Circumf. Area.
As 1 : 12.56637 :: 401.15

401.15
—————
6283185
1256637
1256637
50265480
—————

.
5040.99932550 (70.9990 Root.
49

1409)14099
12681

14189)141893
127701

141989)1419225
1277901

1419989)14132450
12779901

1352549

By Scale and Compasses.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of those Parts will reach from 1 to 71, the Circumference sought.

Problem

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Problem 12. Having the Area, to find the Side of a Square inscribed.

If the Area of a Circle be 1, the Area of a Square inscribed within that Circle will be .6366. Therefore,

$$\text{As } 1 : 401.15 :: .6366$$

$$\begin{array}{r} .6366 \\ \hline 240690 \\ 240690 \\ 120345 \\ 240690 \\ \hline \end{array}$$

255.372090 (1.598 Root, which is the Side of
1 [the Square sought.

$$\begin{array}{r} 25 \overline{) 155} \\ 125 \\ \hline \end{array}$$

$$\begin{array}{r} 309 \overline{) 3037} \\ 2781 \\ \hline \end{array}$$

$$\begin{array}{r} 3188 \overline{) 25620} \\ 25504 \\ \hline \end{array}$$

$$11690$$

The same Reason may be given for the last Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but also all Figures inscribed or circumscribed, have the Squares of their like Sides proportioned to the Circles they are inscribed in, or circumscribed about; and also to the Figures themselves: The Square of any Side of one Figure is to the Area of that

that Figure, as the Square of the like Side of another similar Figure is to the Area thereof; as you may find proved at large in *Euclid*, *Sturmius*, *Mathesis*, *Enucleata*, and other Authors; but will be too large to insert in this Place.

By Scale and Compasses.

Extend the Compasses from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumscribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumscribe that Square, will be 1.4142. Therefore,

$$\text{As } 1 : 1.4142 :: 15.98$$

$$\begin{array}{r} 15.98 \\ \hline 113136 \\ 127278 \\ 70710 \\ 14142 \\ \hline \end{array}$$

22.598916 the Diameter sought.

By Scale and Compasses.

Extend the Compasses from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter sought.

Problem

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Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,
Side Diam. Side of a Square.

As 2 : 1.128 :: 20.0291

1.128

1602328

400582

200291

200291

22.5928248 Diameter.

By Scale and Compasses.

Extend the Compasses from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given) to 22.6, the Diameter of the Circle sought.

Problem 15. Having the Side of a Square, to find the Circumference of a circumscribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompass that Square will be 4.443. Therefore,

Side Sq. Circum. Side Sq.

As 1 : 4.443 :: 15.98

15.98

35544

39987

22215

4443

70.99914 the Circumference.

By Scale and Compasses.

Extend the Compasses from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem

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Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of a Square be 1, the Circumference of a Circle that will be equal thereunto, shall be 3.545. Then,

As 1 : 3.545 :: 20.0291

3.545

1001455

801164

1001455

600873

71.0031595 the Circumference.

By Scale and Compasses.

Extend the Compasses from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference sought.

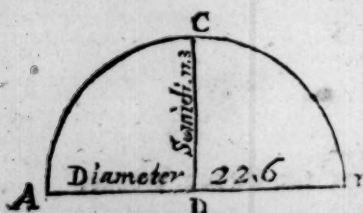
In several of the foregoing Problems, where the Diameter and Circumference are required, the Answers are not exactly the same as the Diameter and Circumference of the given Circle, but are sometimes too much, and sometimes too little, as in the two last Problems, where the Answers in each should be 71, the one being too much, and the other too little. The Reason of this is, the small Defect that happens to be in the Decimal Fractions, they being sometimes too great, and sometimes too little; yet the Defect is so small, that it is needless to calculate them to more Exactness.

§ X. Of a SEMICIRCLE.

TO find the Area of the Semicircle, this is

The R U L E.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch Line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whose Diameter 22.6, and the half Circumference, or Arch Line, ACB, is 35.5, the Half thereof is 17.75, which multiply

by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

17.75 the half Arch Line.
11.3 the Semidiameter.

5325
1775
1775

200.575 the Area of the Semicircle.

By Scale and Compasses.

Extend the Compasses from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may say, by the *Rule of Three*.

As 1 is to .3927, so is the Square of the Diameter to the Area.

By Scale and Compasses.

Extend the Compasses from 1 to the Diameter 22.6; that Extent turned twice from .3927, will reach, at the last, to 200.575.

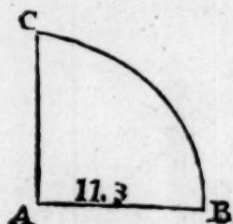


§ XI. *Of a QUADRANT.*

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch Line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle), by the Semidiameter, and the Product is the Area of the Quadrant.



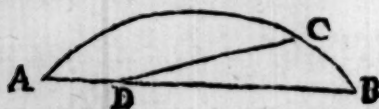
Let ABC be a Quadrant, or fourth Part of a Circle, whose Radius, or Semidiameter, is 11.3, and the half Arch Line 8.875; these multiplied together, the Product is 100.2875 for the Area.

These are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

Before I proceed to shew how to find the Area of the Sector, and Segment of a Circle, I shall shew how to find the Length of the Arch Line, both *Geometrically* and *Arithmetically*.

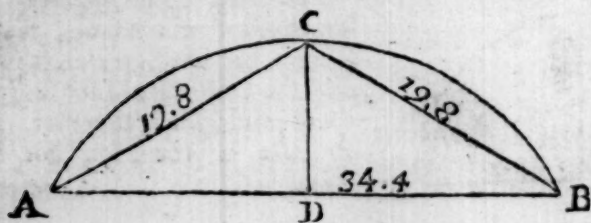
To find the Length of the Arch Line Geometrically.

Divide the Chord Line AB into four equal Parts, and set one of these Parts from B to C, and draw a Line from C to three of those Parts at D, so shall CD be equal to half the Arch Line ACB.



To find the Length of the Arch Line Arithmetically.

Multiply the Chord of half the Segment AC or CB by 8, and from the Product subtract the Chord of the whole Segment AB, and divide the Remainder by 3; the Quotient is the Arch Line ACB sought.



$$\begin{array}{r}
 198 \text{ AC} \\
 \underline{8} \\
 158.4 \\
 34.4 \text{ AB} \\
 \underline{00} \\
 3 \overline{) 124} \\
 \underline{00} \\
 \text{Arch Line } 41.333
 \end{array}$$

Another

Another Way.

From the double Chord of half the Segment's Arch, subtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch Line of the whole Segment.

Thus, if AC 19 8 be doubled, it makes 39 6; from which if you subtract 34.4, the Remainder is 5 2, which, divided by 3, the Quotient is 1.733; this added to 39.6 (the double Chord of the half Segment), the Sum is 41.333. So if the Arch Line ACB was stretched out strait, it would then contain 41.333 such Parts as the Chord AB contains 34 4 of the like Parts.

These two Rules may very easily be proved out of the Table of natural Sines; thus,

Suppose (in the former Figure) the Arch ACB to contain 120 Degrees; the natural Sine of half, *viz.* of 60 Degrees, is 86602; which, being doubled, is 173204, which is the Chord of the whole 120 Degrees, that is, AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the first Rule, multiply 100000 by 8, the Product is 800000; from which subtract 173204 (the Chord AB), and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch Line ACB, according to the first Rule. Now let us examine how near this comes to the true Quantity of the Arch proposed. If the Radius or Semidiameter of a Circle be 100000 (as in the Table of Sines), then the Circumference will be 628318; and because 120 Degrees is the third Part of the Circle, take the third Part of 628318, which is 209439, which is the true Quantity of the Arch ACB in such Parts as

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the Semidiameter contains 100000, and differs from that before found 507, which is a thing inconsiderable in *Practical Mensuration*. The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the same as above; either of the Rules gives the Quantity of the Arch Line too little, and the greater the Arch, the greater the Error. If you know the Degrees that are contained in the Segment's Arch, and would have the Arch Line very exactly, you may reason thus by the *Rule of Three*:

As the Circle's Periphery in Degrees, is to its Periphery in equal Parts; so is the Arch in Degrees and decimal Parts, to the same Arch in equal Parts.

Suppose the Circumference of a Circle be 71, and suppose the Arch to contain 52 Degrees 15 Minutes (the Decimal of 15 Minutes is .25; then say,

Deg. Parts. Deg.
As 360 : 71 :: 52.25

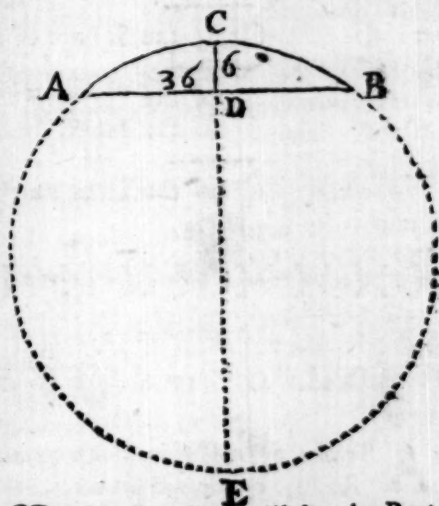
$$\begin{array}{r}
 71 \\
 \hline
 5225 \\
 36575 \\
 \hline
 36 \overline{) 3709} 75 (10.305 \text{ fera.} \\
 \underline{36} \\
 109 \\
 \underline{108} \\
 175
 \end{array}$$

So the 52 Degrees 15 Minutes will contain 10.305 of such Parts as the Circumference contains 71.

Thus have I shewed several Ways of finding the Measure of the Curve Line of any Part of a Circle very near the Truth. The next Thing I shall shew, is,
How

How to find the Diameter of a Circle by having the Chord and versed Sine of the Segment Arithmetically.

Because the Chord AB cuts the Diameter EC at right Angles, therefore the Semichord AD, or DB, is a mean proportional Line between the Parts of the Diameter CD and DE (by *Euc. 6. 13.*); therefore, if you square the Semichord AD, or DB, and divide the Square



by the versed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given versed Sine CD, and the Sum is the Diameter sought.

Example. Let ACB be a Segment given, whose Chord AB is 36, and the versed Sine CD 6; half 36 is 18, which, squared, makes 324; this divided by 6, the Quotient is 54: To which add 6, the Sum is 60, the Diameter of the Circle CE.

18 half the Chord.

18

144

18

6)324 the Square of AD.

54 the Part wanting DE.

6 the versed Sine CD add.

60 the Diameter CE.

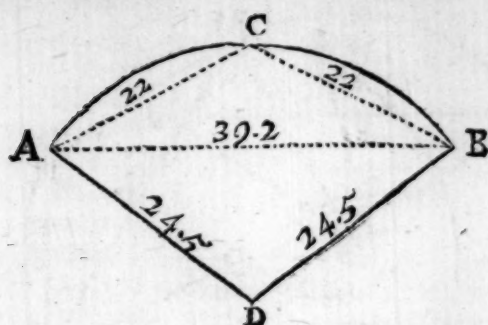
§ XII. *Of the Sector of a CIRCLE.*

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are supposed not to make one Right Line, and a Part of the Circumference: Whence a Sector may be either less or greater than a Semicircle. To find the Area or superficial Content thereof, this is

The R U L E.

Multiply half the Arch Line by the Semidiameter, and the Product is the Area.

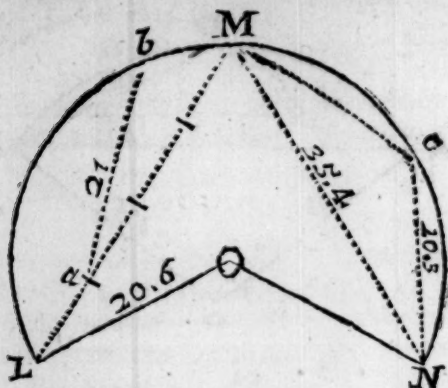
Let ADBC be the Sector of a Circle given, whose Semidiameter AD or BD is 24.5, and the Arch Line ACB (by the first Rule, *Pag.* 112.) I find to be 45.6; the half thereof, 22.8, being multiplied by 24.5 (the Semidiameter) the Product is 558.6; which is the Area of the Sector ACBD.



$$\begin{array}{r}
 22 \\
 8 \\
 \hline
 176 \\
 39.2 \text{ Subtrahend.} \\
 \hline
 3)136.8 \\
 \hline
 45.6 \text{ Arch Line.} \\
 \hline
 22.8 \text{ half.} \\
 24.5 \text{ Semidiam.} \\
 \hline
 1140 \\
 912 \\
 456 \\
 \hline
 558.60 \text{ the Area.}
 \end{array}$$

Again: Let LMNO be a Sector greater than a Semicircle, whose Semidiameter LO or NO is 20.6, and the Line *ba* equal to a fourth Part of the Arch Line *LbMN* 21, the Double whereof is 42, equal to the Arch Line *LbM* or *McN*; or by the arithmetical Rule, *Pag.* 112. the said Arch is found to be 42.333; which, multiplied by 20.6, the Semidiameter, makes 872.0598 for the Area of the Sector LMNO.

See the following Work.



| | |
|------------------|----------------|
| 20.3 Chord Nc. | 42.333 |
| 8 | 20.6 |
| <hr/> | <hr/> |
| 162.4 | 253998 |
| 35.4 Chord MN. | 84666 |
| <hr/> | <hr/> |
| 3) 12.70 | 872.0598 Area. |
| <hr/> | |
| 42.33 Arch Line. | |

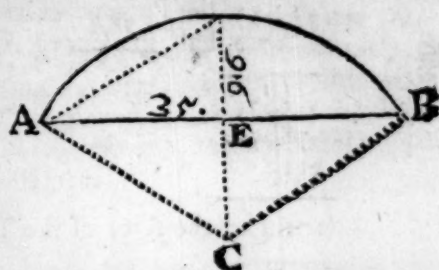
§ XIII. *Of the Segment of a CIRCLE.*

A Segment of a Circle is a Part terminated by a Right Line less than the Diameter, called a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you must, first, find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the last Section), find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and subtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-

Otherwise you may, without describing the Figure, find the Semidiameter of the Circle by the arithmetical Rule (Pag. 113.) and by the arithmetical



Rule (Pag. 112) to find the Arch Line; then multiply half the Arch Line by the Semidiameter; so have you the Area of the Sector: Then subtract the versed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle: Then subtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the Work.

$$2)35 = AB$$

$$17.5$$

$$17.5$$

$$875$$

$$1225$$

$$175$$

$$9.6)306.25(31.9$$

9.6 add.

$$182$$

$$865 \quad 41.5 \text{ the Diameter of the Circle.}$$

$$1 \quad 20.75 \text{ the Semidiameter.}$$

$$9.6 \text{ DE Subtrahend.}$$

$$11.15 \text{ remains the Perpendicular EC.}$$

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11.15 the Perpendicular EC.

17.5 half the Chord AE or EB.

$$\begin{array}{r} 5575 \\ 7805 \\ \hline 1115 \end{array}$$

195.125 the Area of the Triangle.

306.25 the Square of AE.

92.16 the Square of DE the versed Sine.

398.41 Sum.

The Square Root thereof is 19.96 the Chord AD.

159.68
Sub. 35 the Chord AB.

3)124.6

2)41.56 the Arch Line.

20.78 half.

20.75 Semidiameter.

$$\begin{array}{r} 10390 \\ 14546 \\ \hline 4156 \end{array}$$

From 431.1850 Area of the Sect.

Subtract 195.125 Area of the Tri.

Remains 236.060 Area of the Seg.

Again:

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Again: Let MACBM be a Segment greater than a Semicircle; observe the former Rules in all respects, as in the last Example; only, instead of subtracting the Area of the Triangle out of the Area of the Sector, here you must add it thereunto, as may plainly appear by the following Figure.

11.5
8

92.0
20

3)72

24 Half Arch Line.

11.64 Semidiam.
24

4656
2328

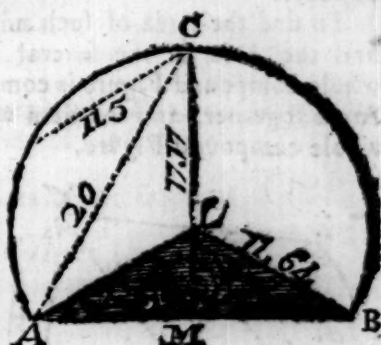
279.36 Area of the Sector LACBL.

10.25 half the Base MA.
5.53 the Perpendicular LM.

3075
5125
5125

56.6825 the Area of the Triangle ALM:
279.36 the Area of the Sector add.

336.0425 the Area of the Segment sought.



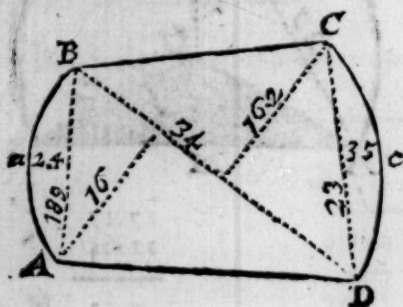
17.17
11.64

5.53 LM.

§ XIV. *Of Compound Figures.*

MIXED or compound Figures are such as are composed of rectilineal and curvilineal Figures together.

To find the Area of such mixed Figures, you must find the Area of the several Figures of which the whole compound Figure is composed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



$$\begin{array}{r} 16.2 \\ 16 \end{array} \left. \vphantom{\begin{array}{r} 16.2 \\ 16 \end{array}} \right\} \text{add.}$$

$$\begin{array}{r} 32.2 \\ \hline \end{array} \text{Sum.}$$

$$\begin{array}{r} 16.1 \text{ half.} \\ 34 \text{ Diagonal.} \end{array}$$

$$\begin{array}{r} 644 \\ 483 \\ \hline \end{array}$$

547.4 Area of the
[Trapezium.]

$$\begin{array}{r} 10.236 \text{ half the Arch Line A a B.} \\ 14.83 \text{ Semidiameter of the Arch AB.} \end{array}$$

$$\begin{array}{r} 30708 \\ 81888 \\ 40944 \\ 10236 \\ \hline \end{array}$$

151.79988 Area of the Sector.

From

Chap. I. *Mensuration of Superficies.* 123

From 14.83 Semidiameter.

Subtract 3.4 Versed Sine.

11.43 Perpend. of the Triangle.

9.45 half the Chord AB.

5715

4572

10287

108.0135 the Area of the Tria. subtracted from

151.7999 the Area of the Sector.

43.7864 the Area of the Segment A a BA.

12.19 half the Arch Line C c D.

20.64 Semidiameter.

4876

7314

24380

251.6016 the Area of the Sector.

From 20.64 the Semidiameter.

Subtract 3.5 Versed Sine.

Rema. 17.14 Perpendicular of the Triangle.

11.5 half the Chord DC.

8570

1714

1714

Sub. 197.110 Area of the Triangle.

From 251.602 Area of the Sector.

Rem. 54.492 the Area of the Segment C c DC.

43.786 the Area of the Segment A a BA.

547.4 the Area of the Trapezium.

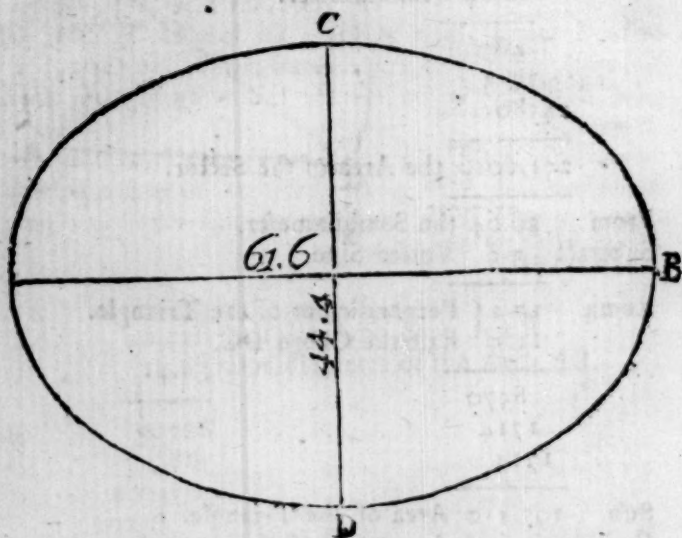
Sum 645.678 the Area of the Whole.

§ XV. Of an ELLIPSIS.

AN Ellipsis, or Oval, is a Figure bounded by a regular Curve Line, returning into itself; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The R U L E.

Multiply the traverse Diameter by the Conjugate, and multiply that Product by .7854, this last Product is the Area of the Oval.



61.6 the traverse Diameter.

44.4 the conjugate Diameter.

2464

2464

2464

2735 04. the Rectangle.

7854 the Area of Unity.

1094016

1367520

2188032

1914528

2148.100416 the Area of the Oval.

Demonstration. If you circumscribe any Ellipsis with a Circle, and suppose an infinite Number of Chord Lines drawn therein, all parallel to the conjugate Diameter, as these in the following Figure; then it will be,

As DA, the Diameter of the Circle, is to N n, the conjugate Diameter of the Ellipsis; so is B a B, any Chord in the Circle, to b a b, its respective Ordinate in the Ellipsis.

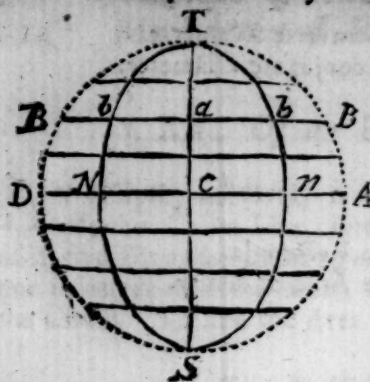
For, according to the Property of the Circle,

| | | |
|----------|---|------------------------------------------------------------|
| it is | 1 | $a S \times T a = \square B a.$ |
| And | | by the Property of the Ellipsis. |
| it is | 2 | $\square TC : \square NC :: a S \times T a : \square b a.$ |
| 1, 2, | 3 | $\square TC : \square NC :: \square B a : \square b a.$ |
| 3, hence | 4 | $TC : NC :: B a : b a.$ |
| Conseq. | 5 | $TC : 2 NC :: 2 B a : 2 b a.$ |
| That is | 6 | $DA : N n :: B a b : b a b.$ |

But the Sum of an infinite Series of such Chords as B a b, do constitute the Area of the Circle. And the Sum of the like Series of their respective Ordinates, as b a b, do constitute the Area of the Ellipsis:

M 3

Thers-



Therefore $TS : Nn ::$ Circle's Area : the Ellipsis Area. But $TS : Nn :: \square TS : TS \times Nn$; whence it follows, that,

$\square TS : \text{Circle's Area} :: TS \times Nn : \text{Ellipsis Area.}$

Consequently, As r is to .7854, so is the Rectangle, or Product, of the transverse and conjugate Diameter of any Ellipsis to its Area.

Hence it is easy to conceive, that the Square Root of the Product of the transverse and conjugate Diameters will be the Diameter of a Circle equal to the Ellipsis.

Hence also the Segments of an Ellipsis, and its circumscribing Circle (whose Bases are parallel to the conjugate Diameter, and of the same Height), are in Proportion one to another as their Bases are. That is,

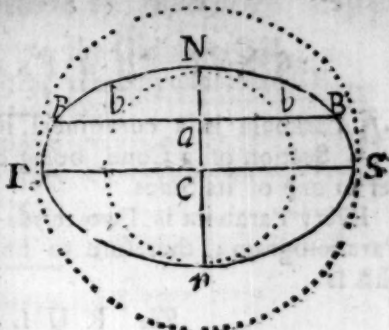
$BaB : b a b :: \text{Area Segment BTB} : \text{Area Segment b T b.}$

Or, $TS : Nn :: \text{Area Segment BTB} : \text{Area Segment b T b.}$

The Area of every Ellipsis is a mean Proportional between the Area of its circumscribing and inscribed Circles.

The Truth of this may easily be deduced from the last; for 'tis already proved, that $\square TS : TS \times Nn ::$ circumscripting Circle's Area : Ellipsis Area.

But $\square TS : TS \times Nn :: TS \times Nn : \square Nn$. Therefore Ellipsis Area : inscribed Circle's Area : $TS \times Nn : \square Nn$.



Example. Let $TS=36$, and $Nn=18.4$. Then $\square TS=1296$, and $\square Nn=338.56$. Then $1296 \times .7854=1017.8784$ great Circle's Area; And $338.56 \times .7854=265.905$, &c. lesser Circle's Area; And $36 \times 18.4=662.4 \times .7854=520.24896$, which is the Area of the Ellipsis; then it will be,

$$1017.878 : 520.24896 :: 520.24896 : 265.905024$$

That is, As the great Circle's Area is to the Area of the Ellipsis, so is the Area of the Ellipsis to the Area of the lesser Circle.

From hence it follows, that all Segments of an Ellipsis, and its inscribed Circle (whose Bases are parallel to the transverse Diameter, and have the same Height) are in Proportion one to another as the Area of the Ellipsis and Circle are.

That is, as the Area of the Circle is to the Area of the Ellipsis, so is the Segment $b N b$: to the Segment BNB ;

Or, $Nn : TS ::$ Area Segment $b N b$: Area Segment BNB .

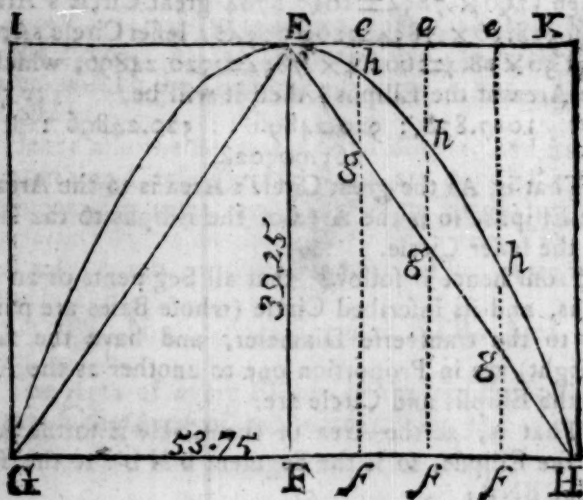
§ XVI. *Of a PARABOLA.*

A Parabola is a curvilinear Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is Two thirds of its circumscribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Base, or greatest Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the last Product by 3, the Quotient will be the Area of the Parabola.



53.75 the Ordinate GH.
39.25 the Perpendicular EF.

$$\begin{array}{r}
 26875 \\
 10750 \\
 48375 \\
 16125 \\
 \hline
 2109.6875 \\
 2 \\
 \hline
 3)4219.3750
 \end{array}$$

1406.4583 the Area.

Demonstration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, &c. and through the Divisions draw Lines, as e f, e f, &c. parallel to the Axis EF. Suppose also EF to be 4.

Then, I say, the Parabolic Sphere Eh HF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, first, g f, g f, g f, &c. are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, f e : g e :: g e : h e; but h e is the Axis EF=0; and in the first Parallel e f must be equal to $\frac{1}{4}$, in the next e f must be equal to $\frac{2}{4}$, in the third to $\frac{3}{4}$, and so on, in a duplicate arithmetical Progression.

For e f (=4) : g e (=1) :: g e (=1) : e h (=1). And the second e f (=4) : e g (=2) :: e g (=2) : e h (=2), &c. And thus it will be, if the Lines F f, f f, &c. be again bisected, &c. *ad infinitum*, so that all the Indivisibles of the trilinear Space EK H h E will be in a duplicate arithmetical Progression increasing. But the Sum of a Rank of such Terms is subtriplicate to a Rank of as many equal to the greatest (by Lemma 3); wherefore the whole trilinear Space EK H h E is to the Parallelogram as 1 to 3; and, consequently, the remaining parabolic Space must be to it as 2 to 3; which was to be proved.

An

130 *Mensuration of Superficies.* Part II.

And since the Triangle FEH is to the Parallelogram as 1 to 2, it must be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4; which was to be proved.

Before I proceed to the Mensuration of solid Bodies, I will lay down such Lemmas as will be necessary to facilitate the Demonstration of all such Solids.

L E M M A I.

In any Series of equal Numbers (representing Lines or other Quantities), as 1, 1, 1, 1, &c. or 2, 2, 2, 2, &c. or 3, 3, 3, 3, &c. if one of the Terms be multiplied into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

L E M M A II.

If a Series of Numbers, in arithmetical Progressions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, &c. (representing a Series of Lines or Roots beginning with a Point) if the last Term be multiplied into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the last Term, N = the Number of Terms, and S = the Sum of all the Series; then will $NL = 2S$; consequently, $\frac{1}{2}NL = S$; viz. One half of so many times the greatest Term as there are Numbers of Terms in the Series.

Thus $\left\{ \begin{array}{l} 0+1+2+3+4=10 \text{ the Sum} = \frac{1}{2}NL. \\ 4+4+4+4+4=20=NL. \end{array} \right.$

L E M M A III.

If a Series of Squares, whose Sides or Roots are in arithmetical Progression, beginning with a Cypher, &c. be infinitely continued, the last Term being multiplied into the Number of Terms, will be triple

to

Instances in cube Numbers.

If 0, 1, 2, 3, 4, 5, &c. be the Roots of the Cubes,

$$1 \left\{ \frac{0+1+8+27}{27+27+27+27} = \frac{36}{108} = \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \right.$$

$$2 \left\{ \frac{0+1+8+27+64}{64+64+64+64+64} = \frac{100}{320} = \frac{5}{16} = \frac{1}{4} + \frac{1}{16} \right.$$

$$3 \left\{ \frac{0+1+8+27+64+125}{125+125+125+125+125+125} = \frac{225}{750} = \frac{3}{10} = \frac{1}{4} + \frac{1}{20} \right.$$

$$(\quad = \frac{1}{10} = \frac{1}{4} = \frac{1}{20}.)$$

From these Examples it plainly appears, that as the Number of Terms in the Series increases, the Fraction or Excess above $\frac{1}{4}$ decreases, the Excess being always $\frac{1}{4N-4}$; which, if we suppose the Series to be infinitely continued, will become infinitely small, or nothing.

LEMMA V.

If a Series of Biquadrates, whose Roots are in arithmetical Progression, beginning with a Cypher, &c. as before, be infinitely continued, the Sum of all the Terms in such a Series will be $\frac{1}{2}$ NLLLL.

The Truth of this may be manifested by the like Process as in the foregoing Lemmas, and so on for higher Powers.

LEMMA VI.

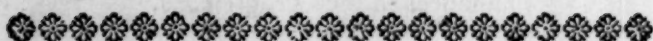
The Sum of an infinite Progression, whose greatest Term is a square Number, the other decreasing by odd Numbers; viz. 1, 3, 4, &c. is in subsesquialteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Instances

Instances in such Progressions.

- 1 $\left\{ \frac{9+8+5}{9+9+9} = \frac{22}{27} = \frac{2}{3} + \frac{4}{27} \right.$
- 2 $\left\{ \frac{16+15+12+7}{16+16+16+16} = \frac{50}{64} = \frac{2}{3} + \frac{15}{64} \right.$
- 3 $\left\{ \frac{25+24+21+16+9}{25+25+25+25+25} = \frac{95}{125} = \frac{2}{3} = \frac{7}{25} \right.$
- 4 $\left\{ \frac{36+35+32+27+20+11}{36+36+36+36+36+36} = \frac{161}{216} = \frac{2}{3} + \frac{17}{216} \right.$

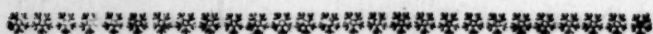
From these Examples it plainly appears, that as the Number of Terms in the Series increases, the Fraction or Excess above $\frac{2}{3}$ decreases; and if we suppose the Series to be infinitely continued, that Excess will quite vanish, and the Sum of the infinite Series will be $\frac{2}{3}$ of so many equal to the greatest.



C H A P II.

The Mensuration of SOLIDS.

SOLID Bodies are such as do consist of Length, Breadth, and Thickness; as Stone, Timber, Globes, Bullets, &c.

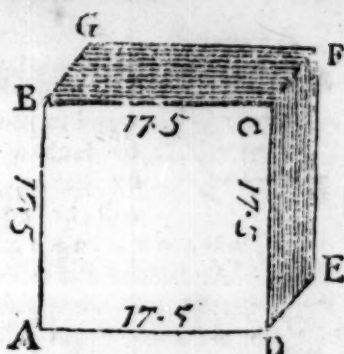
§ I. *Of a CUBE.*

A CUBE is a square Solid, comprehended under six geometrical Squares, being in the Form of a Dye. To find the solid Content, this is

The R U L E.

Multiply the Side of the Cube into itself, and that Product again by the Side; the last Product will be the Solidity, or solid Content of the Cube.

$$\begin{array}{r}
 17.5 \\
 17.5 \\
 \hline
 875 \\
 1225 \\
 175 \\
 \hline
 306.25 \\
 17.5 \\
 \hline
 1531.25 \\
 214375 \\
 30625 \\
 \hline
 \end{array}$$

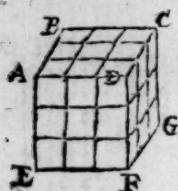


5359-375 the solid Content of the Cube.

Suppose ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17 5 by 17.5, and the Product is 306.25; which being multiplied by 17.5, the last Product is 5359-375, which is 5359 solid Inches and 375 Parts. To reduce the solid Inches to Feet, divide by 1728 (because so many cubical Inches is a Foot), and the solid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compasses.

Extend the Compasses from 1 to 17.5; that Extent turned over twice from 17.5 will reach to 5359, the solid Content in Inches. Then extend the Compasses from 1728 to 1; that Extent, turned the same Way from 5359, will reach to 3 1 Feet.

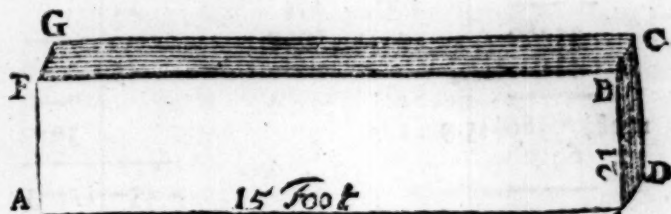


Demonstration. If the Square ABCD be conceived to be moved down the Plane ADEF, always remaining parallel to itself, there will be generated, by such a Motion, a Solid, having six Planes, the two opposite whereof will be equal and parallel to each other; whence it is called a Parallelopipedon, or square Prism. And if the Plane ADEF be a Square equal to the penetrating Plane ABCD, then will the generated Solid be a Cube. From hence such Solids may be conceived to be constituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiplied into the Number of Terms AE, the Product is the Sum of all the Series, (*per Lemma I.*) and, consequently, the Solidity of the Parallelopipedon or Cube. Or, if the Base ABCD, being divided into little square Areas, be multiplied into the Height AE, divided by a like Measure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Base multiplied by the Number of Divisions the Side AE contains. Thus, if the Side of the Base AB be 3, that multiplied into itself is 9, which is the Area of the square Base ABCD; then, if AE be likewise 3, multiply 9 by 3, and the Product is 27; and so many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonstration it is very plain, that, if you multiply the Area of the Base of any Parallelopipedon into its Length or Height, that Product will be the solid Content of such a Solid.

§ II. *Of a PARALLELOPIPEDON.*

LET ABCDEFG be a Parallelopipedon, or square Prism, representing a square Piece of Timber or Stone, each Side of its square Base ABCD being 21 Inches, and its Length AE 15 Feet.



First, then, multiply 21 by 21, the Product is 441, the Area of the Base in Inches; which multiplied by 180, the Length in Inches, and the Product is 79380, the solid Content in Inches. Divide the last Product by 1728, and the Quotient is 45.9, that is, 45 solid Feet and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the same as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet 0 Inches 9 Parts; this multiplied again by 15 Feet, gives 45 Feet 11 Inches 3 Parts, that is, 45 Feet and $\frac{11}{12}$ of a Foot and $\frac{3}{12}$ of $\frac{1}{12}$.

See the Work of all these.

| | | |
|-----------------|-----------------|---------|
| 21 | 44 ¹ | F. I. |
| 21 | 15 | 1—9 |
| — | — | 1—9 |
| 21 | 2205 | — |
| 42 | 44 ¹ | 1—9 |
| — | — | 1—3—9 |
| 44 ¹ | 144)6615(45.9 | — |
| 180 | — | 3—0—9 |
| — | 855 | 15 |
| 35280 | 1350 | — |
| 44 ¹ | — | 45—0—0 |
| — | 54 | 7—6 |
| 1728)79380(45.9 | | 3—9 |
| 6912 | | — |
| — | | 45—11—3 |
| 10260 | | |
| 8640 | | |
| — | | |
| 16200 | | |
| 15552 | | |
| — | | |
| 648 | | |

By Scale and Compasses.

Extend the Compasses from 12 to 21, and that Extent will reach to near 46 Feet, being twice turned over from 15 Feet; so the solid Content is almost 46 Feet.

If the Base of the squared Solid be not an exact Square, but in Form of a rectangle Parallelogram, the Way of measuring it is much the same; for, first, you must find the Area of the Base by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before. Thus,

Chap. 2. *Mensuration of Solids.* 139

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many solid Feet are contained therein?

| | |
|-------------|--------|
| 25 | F. I. |
| 9 | 2—1 |
| <hr/> | 0—9 |
| 225 | <hr/> |
| 25 | 1—6—9 |
| <hr/> | 25 |
| 1125 | <hr/> |
| 450 | 25—0—0 |
| <hr/> | 12—6—0 |
| 144)5625(39 | 1—0—6 |
| 432 | 0—6—3 |
| <hr/> | <hr/> |
| 1305 | 39—0—9 |
| 1296 | |
| <hr/> | |
| 9 | |

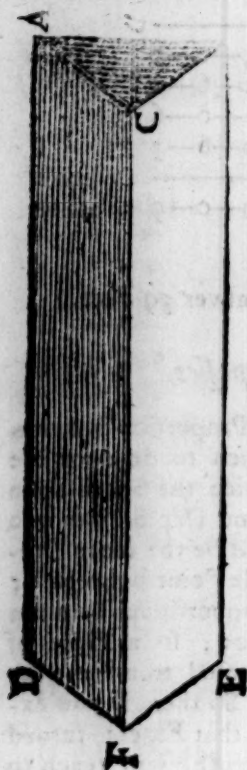
Answer 39 Feet:

By Scale and Compasses.

First, find a mean geometrical Proportion between the Breadth and the Depth; which to do upon the Line of Numbers, you must divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the mean Proportional sought: Thus the middle Point between 25 and 9 is at 15; so is 15 a mean Proportional between 9 and 25, for $9 : 15 :: 15 : 25$; so a Piece of Timber of 15 Inches square is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compasses from 12 to 15, that Extent, turned twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§ III. Of a Triangular PRISM.

A Prism is a Solid contained under several Planes, and having its Bases like, equal, and parallel. The solid Content of a Prism (whether triangular or multangular) is found by multiplying the Area of the Base into the Length or Height, and the Product is the solid Content.



Let ABCDEF be a triangular Prism, each Side of the Base being 15.6 Inches, the Perpendicular thereof Ca is 13.51 Inches, and the Length of the Solid 19.5 Feet

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8 and the Product is 105.378, the Area of the Base; which multiply by the Length 19.5, and the Product is 2054.871; which divide by 144, and the Quotient is 14.27 Feet *ferè*, the solid Content.

| | | | |
|-----------|-------|---------|-------|
| 13 51 | 144 | 2054.87 | 14.27 |
| 7.8 | 144 | | |
| <hr/> | <hr/> | | |
| 10808 | 614 | | |
| 9457 | 576 | | |
| <hr/> | <hr/> | | |
| 105 378 | 388 | | |
| 19 5 | 288 | | |
| <hr/> | <hr/> | | |
| 526890 | 1007 | | |
| 948402 | 1008 | | |
| 105378 | | | |
| <hr/> | | | |
| 2054 8710 | | | |

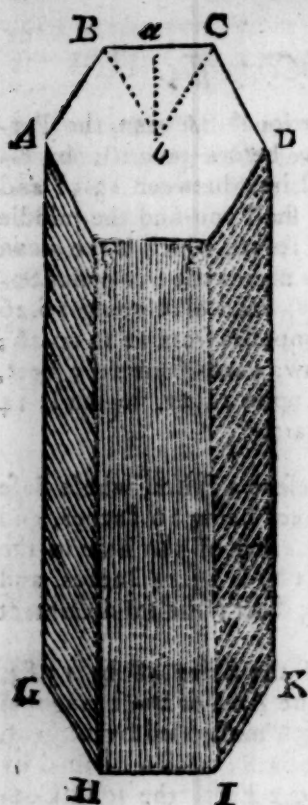
By

By Scale and Compasses.

First, find a mean Proportional between the Perpendicular and Half-side (as before taught), by dividing the Space upon the Line, between 13.51 and 7.8 into two equal Parts; so shall you find the middle Point between them to be at 10.26, which is the mean Proportional sought: By this means the triangular Solid is brought to a square one, each Side being 10.26 Inches. Then extend the Compasses from 12 to 10.26; that Extent, turned twice downwards from 19.5 Feet, the Length will at last fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK represent a Prism, whose Base is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular from the Center of the Base to the Middle of one of the Sides (a b) is 13.14 Inches, and the Length of the Prism is 15 Feet; the solid Content is required.

Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Base (by § VIII. p. 86), which multiplied by 15 Feet, the Length, the Product is 9964.8; which divided by 144, the Quotient will be 69.2 Feet, the solid Content required.



$$\begin{array}{r}
 13.84 \\
 \underline{48} \\
 11072 \\
 \underline{5536} \\
 664.32 \text{ Ar. of Base} \\
 \underline{15} \\
 332160 \\
 \underline{66432} \\
 144)9964.80(69.2 \\
 \underline{864} \\
 1324 \\
 \underline{1296} \\
 288 \\
 \underline{228} \\
 \dots
 \end{array}$$

By Scale and Compasses.

First, find a mean Proportional between the Perpendicular, and half the Sum of the Sides; that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77. Then extend the Compasses from 12 to 25.77; that Extent will reach (being twice turned over) from 15 Feet, the Length, to 69.2 Feet, the Content.

To

Chap. 2. *Mensuration of Solids.* 143

To find the superficial Content of any of the fore-mentioned Solids, you must take the Girth of the Piece, and multiply by the Length, and to that Product add the two Areas of the Bases, the Sum will be the whole superficial Content. Example of the hexagonal Prism last mentioned. The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches; which multiplied by 96, the Product is 17280 square Inches; to which add twice 664.32, the Areas of the two Bases, the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

$$\begin{array}{r}
 180 \\
 96 \\
 \hline
 1080 \\
 1620 \\
 \hline
 17280 \\
 664.32 \\
 664.32 \\
 \hline
 144)18608.64(129.22 \\
 \hline
 420 \\
 1328 \\
 326 \\
 384 \\
 \hline
 96
 \end{array}$$

The superficial Content of the whole Solid is 129.22 Feet.

By Scale and Compasses.

Extend the Compasses from 144 to 180; that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Base, extend the Compasses from 144

to 13.84; that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the superficial Content, as before.

The Demonstration of those last Solids will be the same as in the first Section; for as in that, so in these, the Area of the Base is multiplied into the Length to find the Content, and the same Reason is given for one as for the other.



§ IV. *Of a PYRAMID.*

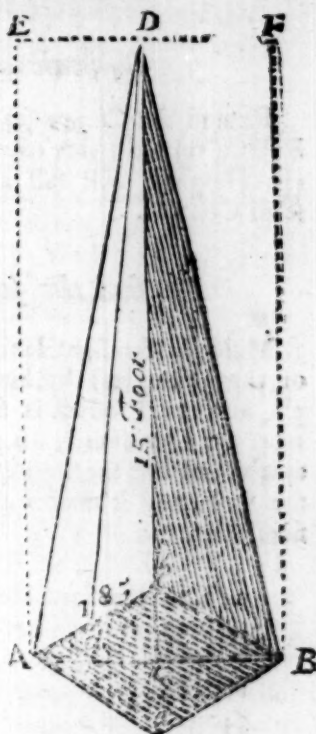
A Pyramid is a solid Figure, whose Base is a Polygon, and whose Sides are plain Triangles, their several Tops meeting together in one Point. To find the solid Content thereof, this is

The RULE.

Multiply the Area of the Base by a third Part of the Altitude, or Length; and the Product is the solid Content of the Pyramid.

Let

Let ABD be a square Pyramid, each Side of the Base being 18.5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Base in Inches; which, multiplied by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the solid Content.



| | |
|--------|-------------------------|
| 18.5 | |
| 18.5 | |
| <hr/> | |
| 925 | |
| 1480 | |
| 185 | |
| <hr/> | |
| 342.25 | Area of the Base. |
| 5 | |
| <hr/> | |
| 144 | 1711.25 (11.88 Content. |

| F. I. Pts. | | | |
|------------|----|---|---|
| 1 | 6 | 6 | |
| 1 | 6 | 6 | |
| <hr/> | | | |
| 1 | 6 | 6 | |
| | 9 | 3 | |
| | | 9 | 3 |
| <hr/> | | | |
| 2 | 4 | 6 | 3 |
| | | | 5 |
| <hr/> | | | |
| 11 | 10 | 7 | 3 |

By Scale and Compasses.

Extend the Compasses from 12 to 18.5 Inches, that Extent, turned twice over from 5 Feet (a third Part of the Height), will fall at last upon 11.88 Feet, the solid Content.

To find the superficial Content.

Multiply the slant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Base 37, and the Product is 6668.88; which divided by 144, the Quotient is 46.31 Feet, the superficial Content of all but the Base; then to that add 2.38 Feet, the Base, and it makes 48.69 Feet, the whole superficial Content.

| | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>180.24 the slant Height d D.</p> <div style="margin-left: 40px;"> <p>37</p> <hr style="width: 100px;"/> <p>126168</p> <p>54072</p> <hr style="width: 100px;"/> </div> <div style="margin-left: 40px;"> <p>844)6668.88(46.31</p> <p>576 2.38</p> <hr style="width: 100px;"/> <p>908</p> <p>864</p> <hr style="width: 100px;"/> <p>448</p> <p>432</p> <hr style="width: 100px;"/> <p>168</p> <p>144</p> <hr style="width: 100px;"/> <p>24</p> </div> | <div style="margin-left: 40px;"> <p>144)342.25(2.38</p> <p>288</p> <hr style="width: 100px;"/> <p>542</p> <p>432</p> <hr style="width: 100px;"/> <p>1105</p> <p>1152</p> <hr style="width: 100px;"/> </div> <p style="margin-left: 40px;">48.69 the whole Content.</p> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

By Scale and Compasses.

Extend the Compasses from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compasses from 144 to 18.5; (one Side of the Base), that Extent will reach from 18.5 to 2.38 *ferè*: Which added to the other, the Sum is 48.69, the whole Superficies.

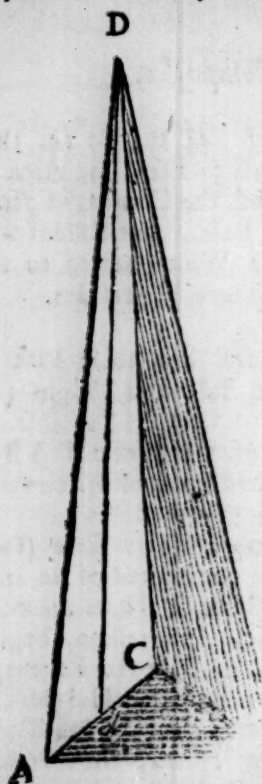
Demonstration. Every Pyramid is a third Part of the Prism, that hath the same Base and Height (by *Eucl* 12.7.)

That is, the solid Content of the Pyramid ABD (in the last Figure) is one third Part of its circumscribing Prism AB EF.

For every Pyramid that hath a square Base (such as Aa Bb in the last Figure) is constituted of an infinite Series of Squares, whose Sides or Roots are continually increasing in arithmetical Progression, beginning at the Vertex or Point D, its Base Aa Bb being the greatest Term, and its perpendicular Height CD is the Number of all the Terms: But the last Term multiplied into the Number of Terms, the Product will be triple the Sum of all the Series (by Lemma 3.);

consequently $\frac{NLL}{3} = S$. And S is equal to the solid

Content of the Pyramid. From hence it will be easy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumscribing Prism (that is, of a Prism of equal Base and Altitude), what Form soever its Base is of; *viz.* whether it be square, triangular, pentangular, &c. You may very easily prove a triangular Pyramid to be a third Part of a Prism of equal Base and Altitude, by cutting a triangular Prism of Cork, and then cut that Prism into three Pyramids, by cutting diagonally, as I have several times done, to satisfy myself and others.



Let ABCD be a triangular Pyramid, each Side of the Base being 21.5 Inches, and its perpendicular Height 16 Feet; the Content, solid and superficial, is required.

First, find the Area of the Base, by multiplying half the Side of the Perpendicular let fall from the Angle of the Base to the opposite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, multiplied by 21.5, the Product is 200.165 Inches, the Area of the Base. Then, because the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52; which divided by 144, the Quotient is 7.41 Feet, the solid Content.

| | | | |
|-------------|-------------------|------------|------------|
| 9.31 | half the Perpend. | F. I. Pts. | |
| 21.5 | the Side. | Side | 1 6 0 |
| | | Half Perp. | 9 4 |
| 4655 | | | |
| 931 | | | 1 4 1 6 |
| 1862 | | | 7 2 |
| 3)200.165 | Area Base. | Area Base | 1 4 8 8 |
| | | | 4 |
| 66.72 | a third Part. | | |
| 16 | Height. | | 5 6 10 8 |
| | | | 4 |
| 40032 | | | |
| 6672 | | | 3)22 3 6 8 |
| 144)1067.52 | (7.41 Solid Cont. | Cont. | 7 5 2 2 |
| 1008 | | | |
| 595 | | | |
| 576 | | | |
| 192 | | | |
| 144 | | | |
| 48 | | | |

In casting this up by Feet and Inches, instead of multiplying by 16, the Height, I break 16 into two such Numbers, as, being multiplied together, the Product may be 16; viz. into 4 and 4, and multiply first by one, and then the other; a third Part of the last Product is the Content.

By Scale and Compasses.

First, find a geometrical mean Proportional (as before directed), by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional sought. Then extend the Compasses from 12 to

O 3

14.15,

14.15, that Extent (turned twice over from 16 Feet) will fall at last upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the superficial Content.

Multiply the slant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Base, and to that Product add the Area of the Base, the Sum is the whole superficial Content.

192.1 Inches, the slant Height d D.

Half Periph. $32.25 = 21.5 + 1075$

$$\begin{array}{r} 9605 \\ 3842 \\ 3842 \\ \hline 5763 \end{array}$$

6195.225 Inches, the Area of all but the
200.165 Area of the Base add. (Base.

144)6395.390(44.41 Feet, the whole Content.

$$\begin{array}{r} 576 \\ \hline \end{array}$$

$$\begin{array}{r} 635 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 593 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 179 \\ 144 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ \hline \end{array}$$

By Scale and Compasses.

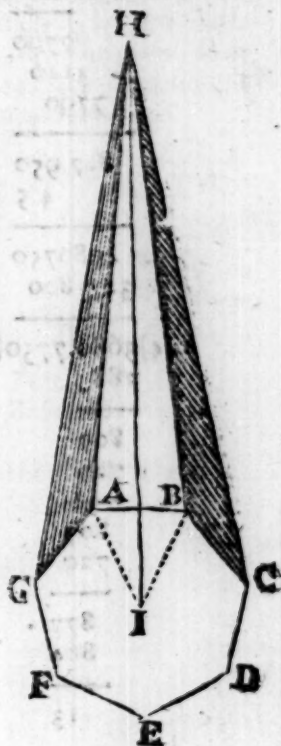
Extend the Compasses from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Base) to 43.02 Feet, the Content of the upper Part.

And

And extend the Compasses from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Base; which, added to the other, makes 44.41 Feet, the Content of the Whole.

Let ABCDEFGH be a Pyramid, whose Base is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the perpendicular Height of the Pyramid HI is 13.5 Feet; the Content, solid and superficial, is required.

Multiply 15.58 (the Perpendicular) by 52.5 (half the Sum of the Sides of the Heptagon) and the Product is 817.95; which multiplied by 4.5, viz. $\frac{1}{3}$ of the Height, and the Product is 3680.775.



Then

Then divide this last Product by 144, and the Quotient is 25.56 Feet, the Content.

15.58 the Heptagon's Perpend.
52.5 the Half-Sum of the Sides.

$$\begin{array}{r} 7790 \\ 3116 \\ \hline 7790 \end{array}$$

817 950
4.5 a third Part of the Height.

$$\begin{array}{r} 4087750 \\ 3271800 \\ \hline \end{array}$$

144)3680.7750(25.56 Solid Feet.
288.

$$\begin{array}{r} 800 \\ 720 \\ \hline \end{array}$$

$$\begin{array}{r} 807 \\ 720 \\ \hline \end{array}$$

$$\begin{array}{r} 877 \\ 864 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \end{array}$$

By Scale and Compasses.

First, find a geometrical mean Proportional between 15.58 and 52.4 (as is before directed), which you will find to be 28.06; then extend the Compasses from 12 to 28.06, that Extent will reach from 4.5 (twice turned over) to 25.56 Feet.

To find the superficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Base 162.75 Inches, by the Half-Sum of the Sides 32.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

| | |
|---------------------------|---------------------------|
| 162.75 | 144)817.95(5.6 |
| 52.5 | |
| <hr/> | |
| 81375 | 979 |
| 32550 | <hr/> 1155 |
| 81375 | <hr/> 3 |
| <hr/> | |
| 144)8544.375(59.335 Feet. | |
| <hr/> 5.68 Base add. | |
| 1344 | |
| 483 | 63.015 the whole Content; |
| 517 | |
| 855 | |
| <hr/> | |
| 135 | |

By Scale and Compasses.

Extend the Compasses from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compasses from 144 to 15.58, the Perpendicular of the Heptagon, that Extent will reach from 5.25 to 5.68 Feet, the Content of the Base; which add to the former, the Sum is 65.015, the whole superficial Content.

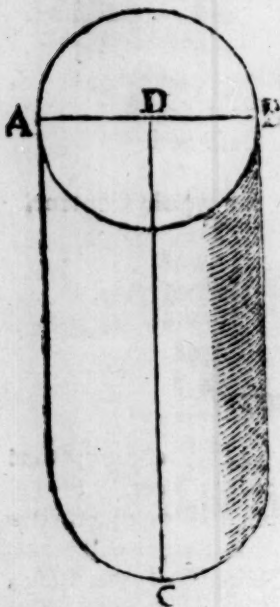
§ V. Of a CYLINDER.

A Cylinder is a round Solid, having its Bases circular, equal, and parallel, in Form of a Rolling-stone used in Gardens. To find the solid Content thereof, this is

The R U L E.

Multiply the Area of the Base by the Length, and the Product is the solid Content.

Let ABC be a Cylinder, whose Diameter AB is 21.5 Inches, and the Length CD is 16 Feet; the solid Content is required.



First, square the Diameter 21.5, and it makes 462.25; which multiplied by 7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this last Product by 144, and the Quotient is 40.34 Feet, the solid Content.

By Scale and Compasses.

Extend the Compasses from 13.54 to 21.5, the Diameter, that Extent (turned twice over from 16, the Length) will at last fall upon 40.34, the solid Content.

To

To find the superficial Content.

First (by Chap. I. Sect. IX. Prob. 2.), find the Circumference of the Base 67.54, which multiplied by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the curve Surface; to which add 504 Feet, the Sum of the two Bases, and the Sum is 95.09 Feet, the whole superficial Content.

| | | |
|--------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|--------|
| $\begin{array}{r} 67.54 \\ \times 16 \\ \hline 40524 \\ 6754 \\ \hline 12) 1080\ 64 \end{array}$ | $\begin{array}{r} 363.05 \\ \times 2 \\ \hline 144) 726.10(5.04 \end{array}$ | |
| $\begin{array}{r} 90.05 \\ 5.04 \\ \hline 95.09 \end{array}$ | $\begin{array}{r} 610 \\ \hline 34 \end{array}$ | } add. |

By Scale and Compasses.

Extend the Compasses from 12 to 67.54 (the Circumference), that Extent will reach from 16 (the Length) to 91.05 Feet, the curve Surface.

And extend the Compasses from 12 to 21.5 (the Diameter), that Extent (turned twice from .7854) will at last fall upon 2.52 Feet, the Area of the Base; which doubled is 5.04; this, added to the curve Surface, makes 95.09 Feet, the whole superficial Content.

Demonstration. The solid Content of every Cylinder is found, by multiplying the Area of its Base into its Height, as aforesaid: For every right Cylinder is only a round Prism, being constituted of an infinite Series of equal Circles; that of its Base, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Base AB, being multiplied into CD,

CD, will be its Solidity (by Lemma I.) Let $D=AB$, $H=CD$.

Then $.7854 DD \times H = \text{its Solidity.}$

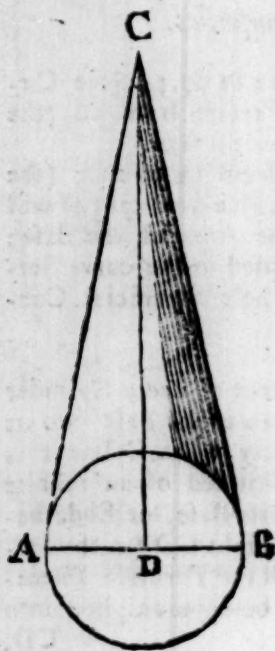


§ VI. Of a CONE.

A Cone is a Solid, having a circular Base, and growing smaller and smaller, till it ends in a Point which is called the Vertex, and may be nearly represented by a Sugar-loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Base by a third Part of the perpendicular Height, and the Product is the solid Content.



Let ABC be a Cone, the Diameter of whose Base AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: First, square the Diameter 26.5, and it is 702.25, which multiply by .7854, and the Product is 551.54715; which multiply by 5.5, and the Product is 3033.47825; which divided by 144, the Quotient is 21.07 *ferè*, the solid Content of the Cone.

26.5 the Diameter.

26.5

1325

1590

530

702.25 the Square:

.7854

280900

351125

561800

491575

551.54 | 715 Area of the Base.
5.5 a third Part of the Height.

275770

275770

144)3033.47|0(21.06 Feet, the Content:

153

947

83

By Scale and Compasses.

Extend the Compasses from 13.54 to 26.5, the Diameter, that Extent turned twice over from 5.5 (a third Part of the Height), it will at last fall upon 21.06 Feet, the Content.

To find the superficial Content.

Multiply half the Circumference 41.626 by the slant Height A C 198.46, and the Product is 8261.09596; which divided by 144, the Quotient is 57.37 *ferè*, the curve Surface; to which add the Base, the Sum is 61.2, the superficial Content.

P

41.626

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41.626 half Circumference of the Base.
198.46 the slant Height.

249756
166504
333008
374634
41626

144)8261.09596(57.37 Feet *ferè*.
3.83 the Base add.
1061
530
989

61.20 the whole Content.

144)551.54(3.83
1195
434
2

By Scale and Compasses.

Extend the Compasses from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the curve Surface.

And extend the Compasses from 12 to 26.5, the Diameter; that Extent, turned twice over from .7854, will at last fall upon 3.83 Feet, the Base; which added to 57.37, the Sum is 61.2 Feet, the superficial Content.

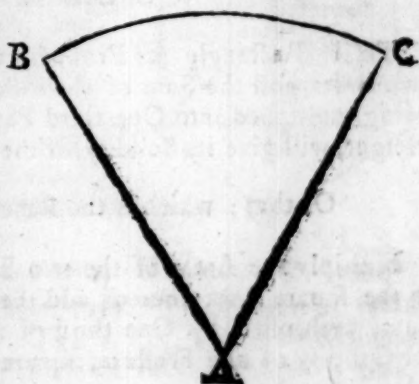
Demonstration. Every Cone is the third Part of a Cylinder of equal Base and Altitude. The Truth of this may easily be conceived, by only considering, that a Cone is but a round Pyramid; and therefore it must needs have the same Ratio to its circumscribing Cylinder, as the square Pyramid hath to its circumscribing

scribing Parallelopipedon; *viz.* as 1 to 3. However, to make it yet clearer, let it be farther considered, That

Every right Cone is constituted of an infinite Series of Circles, whose Diameters do continually increase in arithmetical Progression, beginning at the Vertex, or Point C, the Area of its Base AB being the greatest Term, and its perpendicular Height DC, the Number of all the Terms; therefore the Area of the Circle of the Base, multiplied by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The curve Superficies of every right Cone is equal to half the Rectangle of the Circumference of its Base into the Length of its Side.

For the curve Surface of every right Cone is equal to the Sector of a Circle, whose Arch BC is equal to the Periphery of the Base of the Cone, and Radius AB equal to the slant Side of the Cone: Which will appear very evi-



dent, if you cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend the Sides AB and AC together, till they meet, and you will find it to form a right Cone.

I have omitted the Demonstrations touching the Superficies of all the foregoing Solids, because I thought it needless, they being all composed of Squares, Parallelograms, Triangles, &c. which Figures are all demonstrated before. And if the Area of all such Fi-

gures as compose the Solid, be found severally; and added together, the Sum will be the superficial Content of the Solid.



§ VII. *Of the Frustrum of a PYRAMID.*

A Frustrum of a Pyramid is the remaining Part, when the Top is cut off by a Plane parallel to the Base. To find the solid Content thereof, there are several Rules.

R U L E I.

To the Rectangle (or Product) of the Sides of the two Bases add the Sum of their Squares; that Sum, being multiplied into One-third Part of the Frustrum's Height, will give its Solidity, if the Bases be square.

Or thus; which is the same in Effect:

Multiply the Areas of the two Bases together, and to the square Root thereof add the two Areas; that Sum, multiplied by One-third of the Height, gives the Solidity of any Frustrum, square or multangled.

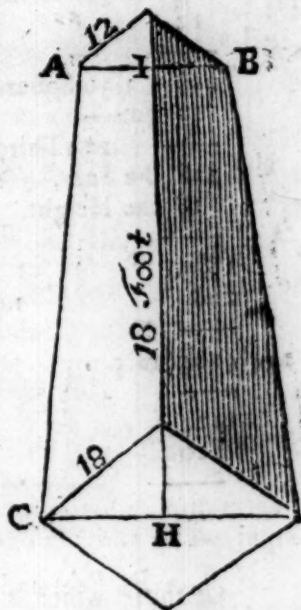
R U L E II.

To the Rectangles of the Sides of the two Bases, add one third Part of the Square of their Difference; that Sum, being multiplied into the Height, will produce the Solidity, if the Bases be Squares: But if they be triangular or multangular, the said Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the square Root thereof will be such a mean Side
as

as will reduce the tapering Solid to a Prism equal thereunto.

Example. Let ABCD be the Frustum of a square Pyramid, the Side of the greater Base 18 Inches, and the Side of the lesser 12 Inches, and the Height 18 Feet; the Solidity thereof is required.

First, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whose Square is 36, a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Base; which multiplied by 18 Feet, the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet, the Content.



Or, by the first Rule, thus: The Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of these three is 684, which multiplied by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the same as before.

See the Work both Ways.

| | | |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} \overbrace{18 \quad 6}^{\text{Diff.}} \\ \underline{12 \quad 6} \\ 216 \quad 3)36 \text{ Square.} \\ \underline{12 \text{ add}} \\ 228 \text{ the Sum.} \\ \underline{18 \text{ the Height.}} \\ 1824 \\ \underline{228} \\ 144)4104(28.5 \\ \underline{1224} \\ 720 \end{array}$ | } | $\begin{array}{r} \overbrace{18 \quad 12}^{\text{Diff.}} \\ \underline{18 \quad 12} \\ 324 \text{ Sq } 144 \text{ Sq.} \\ \underline{144} \\ 216 \\ \underline{684 \text{ the Sum.}} \\ 6 \text{ a 3d of the Height.} \\ 144)4104(28.5 \text{ Feet.} \\ \underline{1224} \\ 720 \end{array}$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

By Feet and Inches, thus:

| | | |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{F. I.} \quad \text{I.} \\ \text{Mult. } 1 \quad 6 \quad 6 \\ \text{by } 1 \quad \quad 6 \\ \hline \text{Prod. } 1 \quad 6 \quad 3)36\text{q.} \\ \text{add } 0 \quad 1 \quad \quad \\ \hline \text{Mult. } 1 \quad 7 \\ \text{by } 18 \quad 0 \text{ Height t.} \\ \hline 18 \quad 0 \\ 9 \quad 0 \\ 1 \quad 6 \\ \hline \text{Cont. } 28 \quad 6 \end{array}$ | } Or thus: { | $\begin{array}{r} \text{F. I.} \\ 2 \quad 3 \text{ Sq. of the greater.} \\ 1 \quad 6 \text{ the Rectangle.} \\ 1 \quad 0 \text{ Square of the less.} \\ \hline 4 \quad 9 \text{ Trip. of a mean Ar.} \\ 6 \quad 0 \text{ a 3d of the Height.} \\ \hline 28 \quad 6 \end{array}$ |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

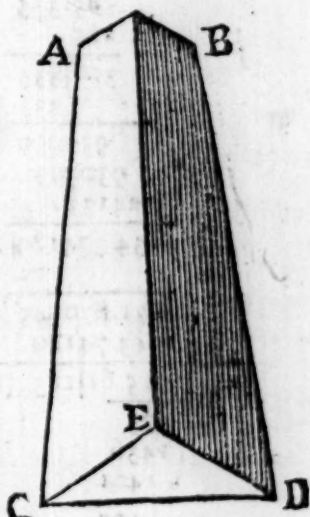
To find the superficial Content.

The Perimeter of the greater Base is 72, and the Perimeter of the lesser Base is 48; add both the Perimeters together, the Sum is 120; the half thereof is 60; which multiplied by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Bases 2.25 Feet, and 1 Foot; the Sum is 93.25 Feet, the whole superficial Content.

| | | |
|-------|-------|------------------------|
| 18 | 12 | 18 the Height. |
| 4 | 4 | 60 |
| <hr/> | <hr/> | <hr/> |
| 72 | 48 | 12)1080 |
| 48 | | <hr/> |
| <hr/> | | 90 Feet. |
| 2)120 | | 2.25 the greater Base. |
| <hr/> | | 1 the lesser Base. |
| 60 | | <hr/> |
| | | 93.25 the Sum. |

Again: Let ABC be the Frustum of a triangular Pyramid, each Side of the greater Base 25 Inches, and each Side of the lesser Base 9 Inches, and the Length 15 Feet; the solid Content thereof is required.

By the second Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16, which squared, makes 256; a third Part thereof is 85.333, which added to 225, the Sum is 310.333; and this multiplied by



433,

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433, the Product is 134.374, &c. which is the Area of a mean Base; and that multiplied by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the first Rule: Find the Area of the greater Base, which you will find to be 270.625, and the Area of the lesser Base will be 35.073; these two Areas multiplied together, the Product is 9491.630625; the Square Root thereof is 97.425; to which add the two Areas, and the Sum is 403.123; which multiplied by a third Part of the Length, 5, the Product is 2015.615; and that divided by 144, the Quotient is 13.99 Feet, as before.

See the Working of both.

$$\begin{array}{r}
 25 \quad 25 \\
 9 \quad 9 \\
 \hline
 \text{Product } 225 \quad 16 \text{ Diff.} \\
 16 \\
 \hline
 96 \\
 16
 \end{array}$$

3)256 the Square.

85.333 a third Part.
225 add.

310.333

433 tabular Number, p. 89.

930999

930999

1241332

134.374189 mean Area.

15 Length.

671.870945

1343.74189

144)2015 61|2835(13.99 Feet.

575

1436

1401

105

Chap. 2. *Mensuration of Solids.*

165

| | | |
|-------------|------------|--------|
| 25 | 9 | 433 |
| 25 | 9 | 81 |
| <hr/> | <hr/> | <hr/> |
| 125 | 81 Square. | 433 |
| 50 | | 3464 |
| <hr/> | | <hr/> |
| 625 Square. | | 35.073 |
| 433 | | |

| | |
|------|---------|
| 1875 | 270.625 |
| 1875 | 35.073 |
| 2500 | <hr/> |

| | |
|--|----------|
| | 811875 |
| | 1894375 |
| | 13531250 |
| | 811875 |

270.625 Area.

9491.630625

9491.630625(97.425

81

187)1391

1309

1944)8263

7776

19482)48706

38964

194845)974225

974225

.....

270 625

270.625 greater Area.

97.425 the mean Proportional.

35.073 the lesser Area.

403.123 the Triple of a mean Area.
5 a third Part of the Height.

144)2015.615(13.99 Feet, the Solidity.

575
1436
1401

105

In finding the Area of the triangular Base, I multiply by .433, because that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Areas, or Multipliers, for finding the Areas of Polygons, you'll find in *p.* 89.

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the superficial Content.

The Perimeter of the greater Base is 75, and the Perimeter of the lesser Base is 27; the Sum of both is 102, and the Half-Sum is 51; which multiplied by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Bases 2.12 Feet, and the Sum is 65.87 Feet, the whole superficial Content,

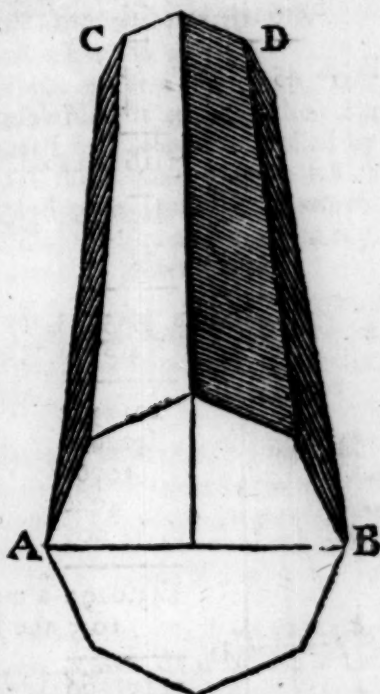
Note, That 51 should have been multiplied by the slant Height, but the Difference it would make is but .06 of a Foot, which is inconsiderable.

Again:

Again : Suppose
A B C D to be the
Frustum of a Pyra-
mid, having an octa-
gonal Base, each Side
thereof being 9 Inches,
and each Side of the
lesser Base 5 Inches,
and the Length, or
Height, 10.5 Feet ;
the Solidity is re-
quired.

By the second Rule
multiply the greater
Side 9 by the lesser
Side 5, and the Pro-
duct is 45 ; then the
Difference between 9
and 5 is 4 ; which
squared makes 16 ; a
third Part thereof is
5.3333, which added
to 45, the Sum is
50.3333 ; multiply
this last by the Number in the Table 4.8284, and the
Product is 243.0292, the Area of a mean Base ;
which multiplied by the Height 10.5 Feet, the Pro-
duct is 2551.8066 ; then divide this last Product by
144, and the Quotient is 17.72 Feet, the solid Con-
tent.

See the Work.



Mult.

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Mult. 9 Inches. 9 from the greater Side.
by 5 Inches. 5 subtract the lesser.

Prod. 45 4
 4

3)16 squared.

5.3333 a third Part.
Add 45

Sum 50.3333 the Sq. of a mean Side.
4.8284 tabular Number, p. 89.

2013332
402666
10067
4026
201

243.0292 a mean Area.
10.5 the Height.

12151460
2430292

144)2551.80|660(17.72
144

1111
1008

1038
1008

300
288

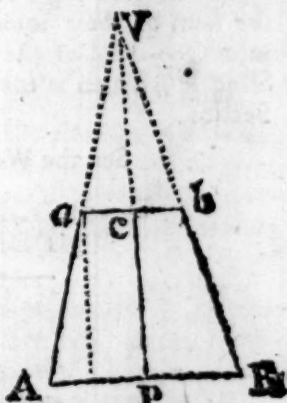
12

To find the superficial Content.

The Perimeter of the greater Base is 72, and the Perimeter of the lesser Base is 40, and their Sum is 112; the Half thereof is 56, which multiplied by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Bases, 3.55, and the Sum is 52.55 Feet, the whole superficial Content.

Demonstration. From the Rules delivered in the IVth and VIth Sections, the two foregoing Rules may easily be demonstrated.

Suppose a square Pyramid, ABV, to be cut by a Plane at $a b$, parallel to its Base AB, and it were required to find the Solidity of the Fruſtum, or Part $a b$ AB. Let there be given



$D = BA$, the Side of the greater Base.
 $d = ba$, the Side of the lesser Base.
 $H = CP$, the perpendicular Height.

First, $D - d : H :: d : \frac{dH}{D - d} = VC$ by the Figure.

Then $DD \times \frac{H + VC}{3} =$ the whole Pyramid BVA,
 by Section the IVth.

And $dd \times \frac{1}{3} VC =$ the Pyramid $a Vb$ cut off.

Q

Then,

To

Then, in the 2d and 3d Steps, if, instead of VC, you take $\frac{d H}{D-d}$ equal to it by the first Step, it will be,

$$\begin{array}{lcl} \text{viz. 1. 2.} & 4 & \frac{DDDH}{3D-3d} = \text{the whole Pyramid BVA;} \\ \text{and 1. 3.} & 5 & \frac{dddH}{3D-3d} = \text{the Pyramid } a V b. \\ & 6 & \frac{DDDH-dddH}{3D-3d} = \text{the Frustum } a b AB. \end{array}$$

And by dividing $DDD-ddd$ by $D-d$, and then multiplying the Quotient by $\frac{1}{3} H$, the last Step will be reduced to $DD+Dd+dd : \times \frac{1}{3} H = \text{the Frustum } a b A B$; which, in Words, is thus:

To the Rectangle of the Sides of the two Bases add the Sum of their Squares; that Sum being multiplied into One-third of the Frustum's Height, will give its Solidity; which is the same as the first Rule of this Section.

See the Work of the Division.

$$\begin{array}{r} D-d) DDD-ddd \quad (DD+Dd+dd \\ \underline{DDD-DDd} \end{array}$$

$$\begin{array}{r} DDd-ddd \\ \underline{DDd-Ddd} \end{array}$$

$$\begin{array}{r} Ddd-ddd \\ \underline{Ddd-ddd} \end{array}$$

o o

The same Reason will hold good for all Frustums of Pyramids or Cones, whether the Base be triangular or multangular, because the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area; which proves the latter Part of the said first Rule.

Again, to prove the second Rule.

Suppose $x = D - d$. And $F =$ the Frustum.

then $DD + Dd + dd = \frac{3F}{H}$ by the last.

1 \odot 2 3 $xx = DD - 2Dd + dd$.

2 — 3 4 $3Dd = \frac{3F}{H} - xx$.

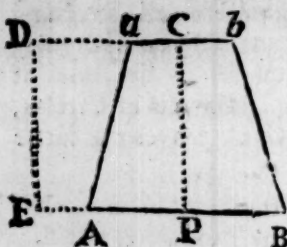
4 — 3 5 $Dd = \frac{F}{H} = \frac{1}{3} xx$. Or $Dd + \frac{1}{3} xx = \frac{F}{H}$

5 $\times H$ 6 $Dd + \frac{1}{3} xx : \times H = F$, the Frustum $a b A b$.

Which, in Words, is thus:

To the Rectangle of the Sides of the two Bases add one third Part of the Square of the Difference of the said Sides, and multiply the Sum of the Height of the Frustum, the Product is the Solidity of the Frustum.

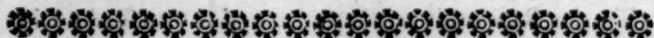
The superficial Contents of Frustums (all but the Bases) are composed of Trapeziums, so many as the Frustum has Sides. As the square Frustum $abAB$, in the last Figure is composed of four Trapeziums, having the two upper, and also the two lower Angles equal; if therefore the Trapezium $abAB$ be cut in two by the Line CP , and the two Pieces laid together, the Line bB upon the Line aA , the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure annexed;



nexed; the Parallelogram DCEP being equal to the Trapezium $abAB$; because the Side Da is equal to PB , and EA is equal to aC . Therefore, to find the Area of the Trapezium, add half the Side ab to half the Side AB , and it makes DC or

EP ; which multiplied by the Height PC , the Product is the Area of the Parallelogram DCEP, equal to the Trapezium $abAB$; then, if that be multiplied by the Number of Trapeziums, the Product will be the superficial Content of the Frustum, wanting the Bases. Or, if the whole Perimeter of the greater Base be added to the Perimeter of the lesser Base, and half the Sum multiplied by the Height, the Product will be the superficial Content of all the Trapeziums at once.

Note, That half the Sum of the Perimeters should be multiplied by the slant Height, up the Middle of one of the Trapeziums; but in the foregoing Examples I have multiplied by the perpendicular Height, because the Difference is very inconsiderable.



§ VIII. Of the Frustum of a CONE.

A Frustum of a Cone, is that Part which remains when the top End is cut off by a Plane parallel to the Base. To find the solid Content, the Rules are the same in Effect as for the Frustum of a Pyramid.

R U L E I.

To the Rectangle of the Diameter of the two Bases add the Squares of the said Diameters, and multiply the

the Sum by .7854, the Product will be the Triple of a mean Area; which multiplied by $\frac{1}{3}$ of the perpendicular Height, that Product will be the solid Content.

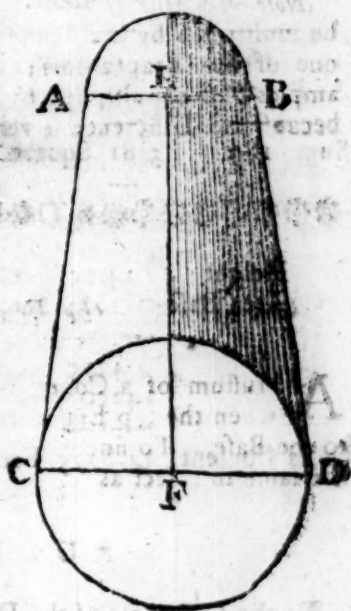
Or thus: Multiply the Areas of the greater and lesser Bases together, and out of the Product extract the Square Root, and add the two Areas and Square Root together, and multiply the Sum by One-third of the perpendicular Height, the Product is the solid Content.

R U L E II.

To the Rectangle of the greater and lesser Diameters, add one-third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiplied by the perpendicular Height, the Product is the Solidity.

Example. Let ABCD be the Frustum of a Cone, whose greater Diameter CD is 18 Inches, and the lesser Diameter AB 9 Inches, and the Length 14.25 Feet, the solid Content is required.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whose Square is 81; a third Part is 67, which add to 162, the Sum is 189; this multiplied by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03



Q 3

Feet.

174 *Mensuration of Solids.* Part II.

Feet, the Area of a mean Base; which multiplied by 14.25 Feet, the Height, the Product is 14.6775 Feet, the solid Content.

Or thus, by the first Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the lesser Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of these three is 567, which multiplied by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Base; this multiplied by 4.75 Feet (a third Part of the Height), and the Product is 14.6775 Feet, the Solidity, the same as before.

See the Work.

| | | |
|----------------------|---------------|-------|
| 18 | 18 from | .7854 |
| 8 | 9 subtr. | 189 |
| <hr/> | <hr/> | <hr/> |
| 162 | 9 Rem. | 70686 |
| Add 27 | 9 | 62832 |
| <hr/> | <hr/> | <hr/> |
| Sum 189 | 3)81 Square. | 7854 |
| | <hr/> | <hr/> |
| | 27 a Third. | 144 |
| | | <hr/> |
| Height 14.25 Feet. | | 444 |
| Area Base 1.03 Feet. | | 432 |
| | | <hr/> |
| | 4275 | 12 |
| | 14250 | |
| | <hr/> | |
| Solid Content | 14.6775 Feet. | |

324 the Square of 18.
162 the Rectangle.
81 the Square of 9.

567 the triple Square of a mean Diameter.

$$\begin{array}{r}
 .7854 \\
 567 \\
 \hline
 54978 \\
 47124 \\
 30270 \\
 \hline
 144)445.32(118 \quad (3.09 \\
 \hline
 1332 \quad 4.75 \\
 \hline
 36 \quad 1545 \\
 \hline
 \quad 2163 \\
 \hline
 \quad 1236 \\
 \hline
 \end{array}$$

The Solidity 14.6775

To find the superficial Content.

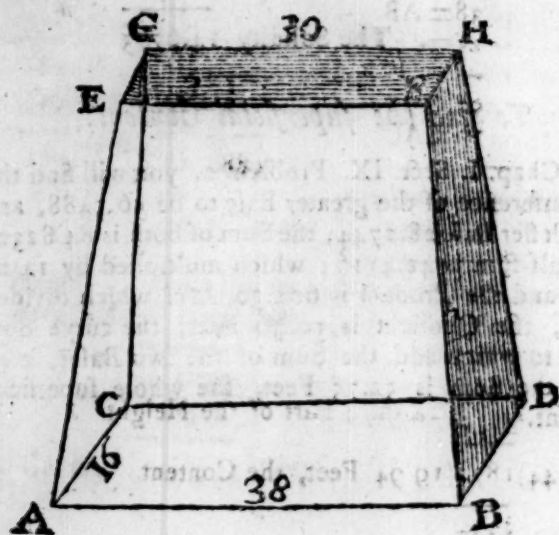
By Chap. I. Sect. IX. Problem 2. you will find the Circumference of the greater Base to be 56.5488, and of the lesser Base 28.2744; the Sum of both is 84.8232; the Half-sum is 42.4116; which multiplied by 14 25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the curve Surface; to which add the Sum of the two Bases, 2.21 Feet, the Sum is 52.75 Feet, the whole superficial Content.

§ IX. To measure the Frustum of a rectangled Pyramid, called a PRISMOID, whose Bases are parallel one to another, but disproportional.

The RULE.

TO the greatest Length add half the lesser Length, and multiply the Sum by the Breadth of the greater Base, and reserve the Product.

Then, to the lesser Length, add half the greater Length, and multiply the Sum by the Breadth of the lesser Base, and add this Product to the other Product reserved, and multiply that Sum by a third Part of the Height, and the Product is the solid Content.



Example.

Example. Let ABCDEFGH be a Prismoid given, the Length of the greater Base AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the lesser Base EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the solid Content is required.

To the greater Length AB 38, add half EF the lesser Length 15, the Sum is 53; which multiplied by 16, the greater Breadth, and the Product is 848; which reserve.

Again, to EF 30, add half AB 19, and the Sum is 49; which multiplied by 12 (the lesser Breadth EG), the Product is 588; to which add 848 (the reserved Product), and the Sum is 1436; which multiplied by 2 (a third Part of the Height), and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the solid Content.

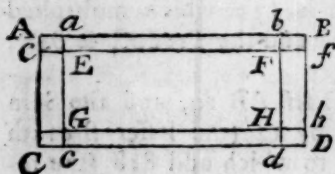
| | |
|----------------------|----------------------|
| 38=AB | 30=EF |
| 15= $\frac{1}{2}$ EF | 19= $\frac{1}{2}$ AB |
| — | —49 |
| 53 | 12=EG |
| 16=AC | — |
| — | 588 |
| 318 | |
| 53 | |
| — | |
| 848 | |
| 588 | |
| — | |
| 1436 | |

2=a third Part of the Height.

144)1872(19.94 Feet, the Content.

$$\begin{array}{r}
 1432 \\
 1360 \\
 640 \\
 \hline
 64
 \end{array}$$

To prove this Rule. Let us suppose the Solid cut into Pieces, so as to make it capable of being measured by the foregoing Rules; thus: Let ABCD represent the greater Base, and EFGH the lesser Base; and let the Solid be supposed to be cut thro' by the Lines, *ac*, *bd*, and *ef*, *gh*, from the Top to the Bottom; so will



there be a Parallelopipedon, having its Bases equal to the lesser Base EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length of the Base by 12, the

Breadth thereof), and the Product is 360; which multiplied by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whose Bases are *abEF*, and *GHcd*; if these two Pieces be laid together, the thick End of one to the thin End of the other, they will compose a rectangled Parallelopipedon; which to measure, multiply the Length of the Base 30 by its Breadth 2, and the Product is 60; which multiplied by 6 (the Height), the Product is 360. Then there are two other Wedge-like Pieces, whose Bases are *eEgG*, and *fFhH*; these two laid together will compose a rectangled Parallelopipedon: To measure this, multiply the Length of the Base 12 by the Breadth 4, the Product is 48; which multiplied by 6 (the Height), the Product is 288. And lastly, there are four rectangled Pyramids, at each Corner one; which to measure, multiply the Length of one of the Bases 4 by its Breadth 2, the Product is 8; which multiplied by 2 (a third Part of the Height) the Product is 16; and that multiplied by 4 (because there are four of them), the Product is 64. Then add all these together, and the Sum is 2872, and divide by 144, the Quotient is 19.94 Feet, the same as before; which shews the Rule to be true.

See the Work.

| | | | |
|-------|-------|-------|-------|
| 12 | 30 | 12 | 4 |
| 30 | 2 | 4 | 2 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 360 | 60 | 48 | 8 |
| 6 | 6 | 6 | 2 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 2160 | 360 | 288 | 16 |
| 360 | | | 4 |
| 288 | | | <hr/> |
| 64 | | | 64 |
| <hr/> | | | |

144) 2872 (19.94 Feet, the whole Content.

1432
1360
640

64

To find the superficial Content.

Half the Perimeter of the greater Base is 54, and half the Perimeter of the lesser Base is 42, which added together, the Sum is 96; which multiplied by 6 (the Height), the Product is 576: Divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Bases 6.72 Feet, and the Sum is 54.72 Feet, the whole superficial Content.

To measure a CYLINDROID; that is, a Frustum of a Cone, having its Bases parallel to each other, but unlike.

The R U L E.

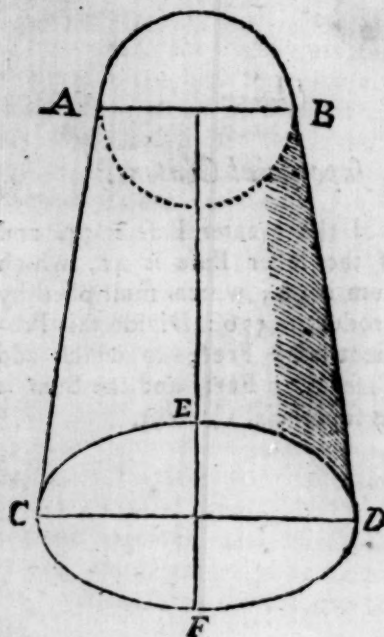
TO the longest Diameter of the greater Base, add half the longest Diameter of the lesser Base, and multiply the Sum by the shortest Diameter of the greater Base, and reserve the Product.

Then, to the longest Diameter of the lesser Base, add half the longest Diameter of the greater Base, and multiply the Sum by the shortest Diameter of the lesser Base, and add the Product of the former reserved Sum, and that Sum will be the triple Square of a mean Diameter; which multiplied by .7854, and

that Product multiplied by a third Part of the Height, the Product is the solid Content.

Exam. Let ABCD be a Cylindroid, whose bottom Base is an Oval, the transverse Diameter being 44 Inches; and the conjugate Diameter 14 Inches; and the upper Base is a Circle, whose Diameter is 26 Inches; and the Height of the Frustum is 9 Feet; the Solidity is required.

To 44 (the greater Diameter of the lower Base) add 13 (half the



the Diameter of the greater Base, the Sum is 57; which multiplied by 14 (the conjugate Diameter of the greater Base, the Product is 798; which reserve. Then to 26 (the Diameter of the lesser Base) add 22 (half the traverse Diameter of the greater Base), and the Sum is 48; which multiplied by 26 (the Diameter of the lesser Base), the Product is 1248; to which add the former reserved Product, the Sum is 2046; which multiplied by .7854, the Product is 1606.9284; which multiplied by 3 (a third Part of the Height), the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the solid Content. See the Work.

| | |
|------------|------------|
| 44=CD | 26=AB |
| 13=half AB | 22=half CD |
| <hr/> | <hr/> |
| 57 Sum. | 48 Sum. |
| 14=E F | 26=AB |
| <hr/> | <hr/> |

| | |
|-------|-------|
| 228 | 288 |
| 57 | 96 |
| <hr/> | <hr/> |

| | |
|-----------------------|----------|
| 798 Product reserved. | 1248 |
| | 798 add. |
| | <hr/> |
| | 2046 |
| | .7854 |
| | 8184 |

| |
|-----------|
| 10230 |
| 16368 |
| <hr/> |
| 14322 |
| <hr/> |
| 1606.9284 |

| |
|----------------------|
| 3 |
| <hr/> |
| 144)4820.78 52(33.47 |

| |
|-------|
| 500 |
| 687 |
| <hr/> |
| 1118 |
| <hr/> |
| 110 |
| R |

This

This Rule being the same as that in the last Section, the Proof of that may serve as a sufficient Proof of this, if what has been before written be well considered.

To find the superficial Content.

To the Periphery of the Ellipsis 97.41, add the Periphery of the Circle 81.68, and the Sum is 179.09; the half thereof 89.545, multiplied by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the curve Surface: Then the Area of the Ellipsis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the curve Surface, the Sum is 74.2 Feet, the whole superficial Content.



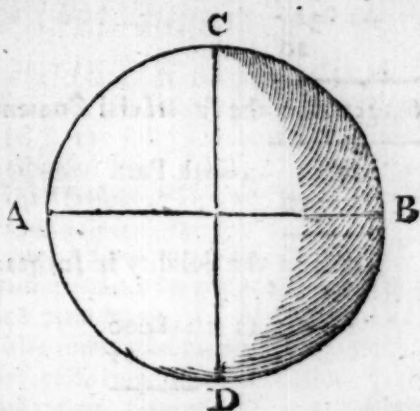
§ XI. *Of a SPHERE or GLOBE.*

A Sphere, or Globe, is a round solid Body, every Part of whose Surface is equally distant from a Point within it, called its Center; and it may be conceived to be formed by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

The R U L E.

1. Multiply the Axis, or Diameter, into the Circumference, the Product is the superficial Content; which multiplied by a sixth Part of the Axis, the Product is the Solidity.
2. Or thus: As 21 is to 11, so is the Cube of the Axis to the solid Content.
3. Or, as 1 is to .5236, so is the Cube of the Axis to the solid Content.

Example. Let ABCD be a Globe whose Axis is 20 Inches, then the Circumf. will be 62.832 : Then, by the first Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the superficial Content in Inches;



take a sixth Part thereof, which is 209.44, (because an exact sixth Part of 20 cannot be taken), multiply that sixth Part by 20 (the Axis), and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the superficial Content by the Axis, and take a sixth Part of the Product, the Answer will be the same.

Or thus, by the second Rule :

The Cube of the Axis is 8000; which multiplied by 11, the Product is 88000; which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule :

If the Cube of the Axis be multiplied by .5236, the Product is 4188.8, the Solidity, the same as by the first Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

62.832
20

6)1256.640 the superficial Content.

209.44 a sixth Part.
20

4188.80 the Solidity in Inches.

21 : 11 :: 8000
11

21)88000(4190.47 the Content.

40
190
100
160
13

: .5236 :: 8000
8000

1728)4188.8000(2.424 Feet, the Solidity.

7328
4160
7040
128

Note, If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

By Scale and Compasses.

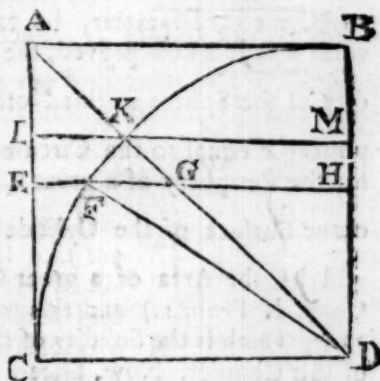
Extend the Compasses from 1 to 20 (the Axis), that Extent (turned three times over from .5236), will at the last fall upon 4188.8, the solid Content in Inches : Or, extend the Compasses from 1728 to 8000 (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the solid Content in Feet.

Extend the Compasses from 1 to 20 (the Axis,) that Extent (turned twice over from 3.1416), will at last fall upon 1256.64, the superficial Content in Inches : Or, extend the Compasses from 144 to 400 (the Square of the Axis), that Extent will reach from 3.1416 to 8.72, the superficial Content in Feet.

Demonstration. Every Sphere is equal to a Cone, whose perpendicular Axis is the Radius of the Sphere, and its Base a Plane equal to all the Surface of it.

For you may conceive the Sphere to consist of an infinite Number of Cones, whose Bases, taken altogether, compose the Surface, and whose Vertexes meet altogether in the Center of the Sphere : Hence the Solidity of the Sphere will be gained, by multiplying its Surface by $\frac{1}{3}$ of its Radius.

Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be supposed all three to revolve round the Line BD as an Axis : Then will the Square generate a Cylinder, the Quadrant a Hemisphere, and the Triangle a Cone, all of the same Base and Altitude.



R 3

Then

Then the Square of EH ($= \square FD$) $= \square FH + \square DH$ (but $DH = GH$). And since Circles are as the Squares of their Diameters (by *Euclid* 12. 2.) the Circle made by the Revolution of EH must be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring described by the Motion of EF. And thus it will always be, wherever you draw the Line EH or IM, &c.

Therefore the Aggregate, or Sum, of all the Rings, made by the Revolution of the EF's, must be equal to that of all the Circles made by the Motion of the GH's; *i. e.* the Dish-like Solid, formed by the revolving Rings, will be equal to the Cone, formed by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Dish-like Solid will be as the Cone, $\frac{1}{3}$ of the circumscribing Cylinder, and consequently the Hemisphere must be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder.

Let the Radius of the Sphere be $r = CD$, then the Diameter will be $2r$, let the Surface of the Sphere, generated by the revolving Semicircle, be called S , and that of the Cylinder, formed by the Revolution of $2AC = 2r = \text{Diameter}$, be called f . Wherefore in what was just now proved, the Expression for the Solidity of the Sphere in this Notation will be $\frac{1}{3} \frac{rS}{c}$; and putting c equal to the Circumference of the Base, or for the Periphery of a great Circle of the Sphere, the curve Surface of the Cylinder will be $2rc$, also $\frac{rc}{2}$ will be the Area of a great Circle (by Sect. IX. of Chap. I. Prob. 1.) and this multiplied by 21, makes $11c$; which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, since f was put equal to $2rc =$ the

the curve Surface of the Cylinder $\frac{f r}{2}$ (by substituting f for $2 r c$) will be also = the Solidity of the Cylinder. Now, since the Sphere is $= \frac{2}{3}$ of the Cylinder, $\frac{r S}{3} = \frac{2 \times f r}{3 \times 2}$; that is, $\frac{r S}{3} = \frac{2 f r}{6} = \frac{f r}{3}$. Wherefore $r S = r f$, that is, dividing by r , $S = f$; or the Surface of the Sphere is equal to the curve Surface of the Cylinder, but the curve Surface of the Cylinder was $2 r c$.

Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you must multiply the Diameter ($= 2 r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Base. From this Notation also $\frac{r c}{2}$, the Area of a great Circle of the Sphere is plainly $\frac{1}{4}$ of $2 r c$, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greatest Circle of it.

Wherefore, to $2 r c$, the convex Surface of the Cylinder, add $r c$, equal to the Area of both its Bases, you will have $3 r c$; which shews you, that the Surface of the Cylinder (including its Cases) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the same Base and Altitude, by Lemma VI. aforegoing, thus:

Let AGB represent the Hemisphere, and AIKB half the Cylinder; then, if the Semidiameter GH be divided into fix equal Parts, and Lines drawn parallel to AB, the Diameter,



the Squares of the Semichords, $ab, cd, ef, \&c.$ will be

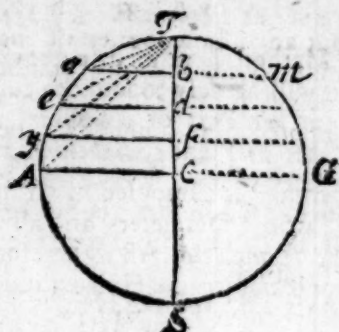
be a Series of Numbers, whose greatest Term AH is a square Number, the other differing by odd Numbers; that is, AH is 36, kl 35, gh 32, ef 27, cd 20, ab 11: But an infinite Series of such Numbers are in Proportion to the infinite Number of Terms equal to the greatest as 2 to 3. And because the Hemisphere is composed of an infinite Number of Circles, whose Diameters are the Chords of the Semicircle; and the Half-cylinder is composed of an infinite Number of Circles, whose Diameters are all equal to the Diameter of the Semicircles AB; therefore the Hemisphere is in Proportion to the Half-cylinder as 2 to 3; and consequently the whole Sphere bears the same Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greatest Circle, is thus proved:

The Solidity of the Sphere is constituted of an infinite Number of parallel Circles (as is aforesaid); consequently the Superficies of the Sphere will be composed of the Peripheries of those Circles which constitute its Solidity.

Note, In the following Demonstration, \odot signifies any Circle in general; and if any two Letters be joined to it, thus, \odot AB, &c. then it denotes the Area of such a Circle as those two Letters represent the Radius of.

Let



Let $D=TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be

that is,

therefore

For

and

{

$$1 \quad D - Tb \times Tb = \square ab;$$

$$2 \quad D \times Tb = \square Tb = \square ab;$$

$$3 \quad D \times Tb = \square aT.$$

$$4 \quad \square AB + \square Tb = \square aT \text{ (Eucl. I. 47.)}$$

$$5 \quad D \times DT = \square eT.$$

$$6 \quad D \times Tf = \square yT.$$

Hence it is evident, that the Series $\square aT$, $\square eT$, $\square yT$, &c. are in the same Ratio with Tb , Td , Tf , &c. viz. in arithmetical Progression: Whence it follows, that the $\odot aT$ is to the Sum of all the Circles Peripheries between T and b .

And $\odot eT$ is the Sum of all the Circles Peripheries between T and d , &c.

Consequently, that the $\odot AT$ is the Sum of all the Circles Peripheries, included between T and C ; that is, $\odot AT$ is the Superficies of the Hemisphere.

And because $\square AC + \square TC = \square AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT = 2 \odot AC$, is the Superficies of the Hemisphere.

Consequently, $4 \odot AC$ will be the Superficies of the whole Sphere. Which was to be proved.

Scholium.

Scholium.

From the Method here used in proving the whole Superficies, it will be easy to find the curve Superficies of any Frustrum, or Part of a Sphere, that is cut off by a Right Line, or Plane; viz. such as the Frustrum aTm in the last Scheme, whose curve Superficies is $\odot aT$, as above. Therefore (because $\square ab + \square Tb = \square aT$) it will be $\odot ab + \odot Tb =$ the curve Superficies of that Frustrum.

But if the Axis TS, and the Height Tb of the Frustrum, are given, then it will be $TS \times Tb = \square aT$, as in the third Step above; which gives the Proportion or Theorem following; viz.

As the Axis of the Sphere is to the whole Superficies of the Sphere; so is the Height of any Frustrum to its curve Superficies.

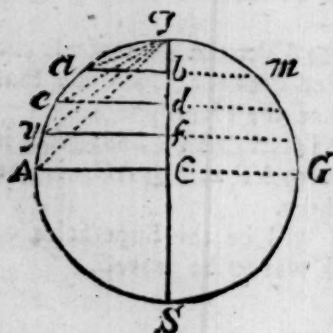
To which if there be added the Area of the Frustrum's Base, the Sum will be the whole Superficies of the Frustrum.

That the Solidity of every Sphere is Two-thirds of its circumscribing Cylinder, may be thus proved.

According to the Work above, it appears, that $\odot ab$, $\odot ed$, $\odot yf$, &c. do constitute the Solidity of the Sphere; and that $\square aT$, $\square eT$, $\square yT$, &c. are

a Series of Terms in arithmetical Progression, $\square AT$ being the greatest Term, and TC the Number of Terms; therefore $\odot AT \times \frac{1}{2} TC =$ the Sum of all the Series, by Lemma 2.

And because $\square aT = \square Tb + \square ab$. $\square aT = \square Td + \square ed$. $\square yT = \square Tf + \square yf$. $\square AT = \square TC = \square AC$, &c. wherein



wherein $\square Tb$, $\square Td$, $\square Tf$, &c. are a Series of Squares, whose Roots Tb , Td , Tf , are in arithmetical Progression; $\square TC$ being the greatest Term, and TC the Number of Terms; therefore $\odot TC \times \frac{1}{3} TC =$ the Sum of all the Series, by Lemma III.

Consequently, $\odot AT \times \frac{1}{2} TC - \odot TC \times \frac{1}{3} TC =$ the Sum of all the Series $\odot ab$, $\odot ed$, $\odot yf$, &c. which constitute the Solidity of the Half-sphere ATG . Put $D=2TC$, the Axis of the Sphere; then $\frac{1}{3} D = \frac{1}{3} TC$, and $\frac{1}{6} D = \frac{1}{6} TC$. And because $\square AT = 2 \square TC$, therefore $\odot AT = 2 \odot TC = 1.5708 DD$; and $1.5708 DD \times \frac{1}{4} D = 0.3927 DDD$.

Again; $\odot TC \times \frac{1}{3} TC = 0.7854 DD \times \frac{1}{6} D = .1309 DDD$, then $0.3627 DDD - 0.1309 DDD = 0.2618 DDD$, the Solidity of the Half-sphere.

Consequently, $0.2618 DDD \times 2 = .5236 DDD$ will be the solid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whose Base and Height, is $=D$.

For $0.7854 DDD =$ the Solidity of the Cylinder, by Sect. V. But $\frac{2}{3}$ of $0.7854 DDD = 0.5236 DDD$, as before.

Scholium.

From this Demonstration it will be easy to deduce or raise Theorems for finding the solid Content of any Frustum of a Sphere, as Tm in the last Figure.

For we there suppose the Frustum aTm to be constituted of an infinite Series of Circles, which have the same Ratio with all those Circles that constitute the Half-sphere.

Therefore it follows, that $\odot aT \times \frac{1}{2} TB$; $- \odot bT \times \frac{1}{3} Tb$, will be the Sum of all the Circles intercepted between T and b ; consequently it will be the Solidity of that Frustum.

And, because $\square ab + \square Tb = aT$; therefore $\odot ab + \odot Tb \times \frac{1}{2} Tb$; $- \odot Tb \times \frac{1}{3} Tb =$ the Solidity. Let $c=ab$ half the Diameter of the Frustum's Base $h=Tb$ its Height; and $S=$ the Solidity of the Frustum. Then $\odot ab = 3.141600$, and $\odot Tb = 3.1416 hh$; consequently,

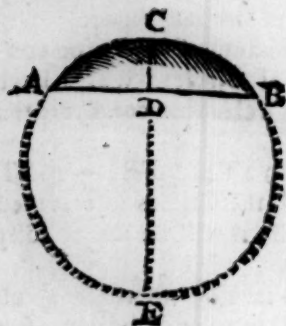
sequently, $\frac{3.1416cch + 3.1416hhh}{2} - \frac{3.1416hhh}{3} = S.$

Which, being reduced, will become $3 cch + hhh \times 0.5236 = S$; which is one Theorem for finding the Solidity of the Frustum, and may be expressed in Words, thus:

If to three times the Square of the Semidiameter of the Frustum's Base, you add the Square of the Height of the Frustum, and multiply the Sum by the Height of the Frustum, and that Product multiplied by .5236, the Product will be the solid Content.

But if the Axis of the Sphere, and the Height of the Frustum, be given; then put $D =$ the Axis, $h =$ the Height of the Frustum, and c as before; it will be $D - h \times h = cc$, viz. $Dh - hh = cc$. Then will $3 Dh - 2hhh = 3cch + hhh$; consequently, $3 Dh - 2hhh \times 0.5236 = S$, the Frustum's Solidity: Which is another Theorem for finding the Solidity of the Frustum, and may be expressed in Words, thus:

From three times the Axis subtract twice the Height of the Frustum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this last Product will be the Solidity of the Frustum.



Example. Let ABCD be the Frustum of a Sphere; suppose AB (the Diameter of the Frustum's Base) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By

By the first Rule.

$$\begin{array}{r}
 8 \\
 8 \\
 \hline
 64 \text{ Square of the Semidiameter AD.} \\
 3 \\
 \hline
 192 \\
 16 \text{ add the Square of CD.} \\
 \hline
 208 \\
 4 \text{ multiply by CD.} \\
 \hline
 832
 \end{array}$$

$$\begin{array}{r}
 .5236 \\
 832 \\
 \hline
 10472 \\
 15708 \\
 41888 \\
 \hline
 435.6352
 \end{array}$$

By the second Rule, thus:

First, by the Rule in Page 113, you will find the Axis of the whole Globe to be 20 Inches.

| | | | |
|--------|---------------|----------|---------------------------------------------|
| | 20 Axis. | .5236 | |
| | 3 | 832 | |
| | <hr/> | | |
| From | 60 | 10472 | |
| Subtr. | 8 twice CD. | 15708 | |
| | <hr/> | 41888 | |
| Rem. | 52 | <hr/> | |
| Mult. | 16 Sq. of CD. | 435.6352 | } the solid Content, the same as before. |
| | <hr/> | | |
| | 312 | | |
| | 52 | | |
| | <hr/> | | |
| Prod. | 832 | | |

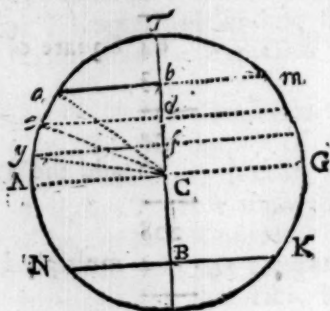
S

I

And,

And, if it be required to find the middle Part, amNK, usually called the middle Zone of a Sphere.

Then, because it is supposed that $am = NK$, or (which is all one) that $bC = CB$; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone amNK.



But because the Work is a little troublesome, I will here shew how to raise a Theorem for the doing it:

First, because $AC = yC = eC = aC = TC$; therefore it will be $\square AC - \square Cf = yf$, $\square AC - \square CD = ed$, $\square AC - \square Cb = ab$, &c.

Here because $\square AC$, $\square AC$, $\square AC$, &c. are of Series of Equals, and Cb the Number of all the Terms; therefore $\square AC \times Cb =$ the Sum of all that Series (*per* Lemma I.)

And $\square Cf$, $\square Cd$, $\square Cb$, &c. being a Series of Squares, whose Roots are in arithmetical Progression, beginning at the Center, C; viz. o, Cf, Cd, Cb, &c. wherein the greatest Term is $\square Cb$, and the Number of Terms is Cb; therefore $\square Cb \times \frac{1}{3} Cb =$ the Sum of all the Series (*per* Lemma III.)

Consequently, the $\odot AC \times Cb : - \odot Cb \times \frac{1}{3} Cb =$ the Sum of all the Series $\odot yf$, $\odot ed$, $\odot ab$, &c. which do constitute the Solidity of the half Zone amAG.

And because $\square AC - \square Cb = \square ab$; therefore
 $\odot AC - \odot ab = \odot Cb$. Consequently $\odot AC \times Cb$:
 $\frac{\odot AC + \odot ab : \times Cb}{3} = 2 \odot AC + \odot ab : \times \frac{1}{3} Cb$
 will be the Solidity of the half Zone.

Put $D = AG = 2 AC$, $x = am$, and $H = bB = 2 Cb$.

Then $\odot AC = .7854 DD$, $\odot ab = .7854 xx$. And if we turn the common Factor .7854 into a Divisor 1.27323, and then take the triple of that Divisor; viz. 3.8197, the Result of the precedent Work will produce the following Theorem.

$$\text{Theo. } \left\{ \frac{2DD + xx}{3.8197} : \times H = \right\} \text{ the middle Zone } amNK.$$

Which in Words is thus: To twice the Square of the Axis AG , and the Square of the Diameter of the Frustum's Base (am), divide the Sum by 3.8197, then multiply the Quotient by the Height or Thickness of the middle Zone, and the Product will be the Solidity of the middle Zone required.

This is so plain and easy, that it needs no Example.

§ XII. Of a SPHEROID.

A Spheroid is a Solid resembling an Egg. To find the solid Content thereof, this is

The R U L E.

Multiply the Square of the Diameter of the greatest Circle by the Length, and that Product multiply again by .5236; this last Product will be the Solidity of the Spheroid.



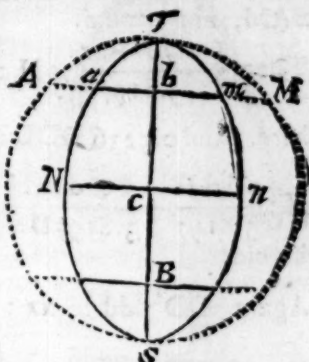
Let AB, the Diameter of the greatest Circle, be 33 Inches, and CD (the Length) 55 Inches, the Solidity is required.

| |
|-------|
| 33 |
| 33 |
| — |
| 99 |
| 99 |
| — |
| 1089 |
| 55 |
| — |
| 5445 |
| 5445 |
| — |
| 59895 |

| |
|--------------------------|
| 59895 |
| .5236 |
| — |
| 359370 |
| 179685 |
| 119790 |
| 299475 |
| — |
| 31361.0220 the Solidity. |

Demonstration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whose Base is equal to the greatest Circle of the Spheroid, and its Height equal to the Length of the Spheroid.

Suppose the Figure NTnSN, in the annexed Scheme, to represent a Spheroid, formed by the Rotation of the Semi-Ellipsis TNS, about its transverse Axis TS.



Let $D = TS$, the Length of the Spheroid, and the Axis of the circumscribing Sphere; and $d = Nn$, the Diameter of the greatest Circle of the Spheroid:

Then, because $\square TC : \square NC :: \square Ab : \square ab$, by *Secl. XV. Step 3. Page 125.*

Therefore it will be, $DD : dd :: \square Ab : \square ab$.

But the Sum of an infinite Series of such Circles as $\odot Ab$ (whose Diameters are Chords) do constitute the Solidity of the Sphere. (By *Secl. XI.*)

And the Sum of an infinite Series of such Circles as $\odot ab$ (*viz.* whose Diameters are Ordinates of the Ellipsis) do constitute the Solidity of the Spheroid.

Therefore, $DD : dd :: 0.5236 DDD : 0.5236 Ddd =$ the Solidity of the Spheroid. (*Eucl. 5. 12.*)

But $0.5236 Ddd = \frac{2}{3}$ of the Cylinder, whose Diameter is $= d$, and Height $= D$. (By *Secl. V.*)

Now, from this Proportion, between the Sphere and its inscribed Spheroid, it will be very easy to deduce Theorms for finding the solid Content, either of the Frustum or middle Zone of any Spheroid; having the same Height with that of the Sphere; for,

As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid, so is any Part of the Sphere to the like Part of the Spheroid.

As for Instance : Suppose it was required to find the middle Zone of any Spheroid.

Let $D=TS$, and $d=Nn$, as above; and $H=bB$, $x=AM$, and $c=am$.

Then $\left\{ \frac{2DD+xx}{3.8197} \times H = \text{the middle Zone of the Sphere. And } 0.5236 DDD : 0.5236 ddD :: \frac{2DD+xx}{3.8197} \times H : \frac{2ddH}{3.8197} + \frac{xxdH}{3.8197DD} = \text{the middle Zone of the Spheroid.} \right.$

Again, $DD : dd :: xx : cc$. Therefore $\frac{xxdd}{DD} = cc$.

Consequently, $\frac{xxdd}{DD} \times \frac{H}{3.8197} = \frac{cc}{3.8197} \times H$. Which being taken instead of $\frac{xxdH}{3.8197DD}$, there will arise the following Theorem $\left\{ \frac{2dd+cc}{3.8197} : \times H = \text{the middle Zone of the Spheroid.} \right.$

Note, That $3.8197 = 1.2732 \times 3$. See Page 102.



§ XIII. Of a Parabolic CONOID.

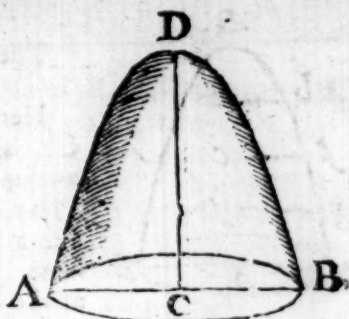
A Parabolic Conoid is something like a half Spheroid, having its Sides somewhat flatter. It is generated by supposing a Semi-parabola turned about its Axis. To find the solid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Base by .7854, and multiply that Product by half the Height, that last Product shall be the solid Content.

Let

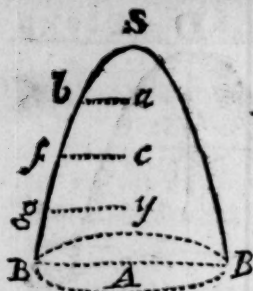
Let ABCD be a Parabolic Conoid, the Diameter of whose Base is 36 Inches, and its Height CD 33 Inches; the Solidity is required.



| | | |
|------|------------------------------------------|--------------|
| 36 | .7854 | 1017.8784 |
| 36 | 1296 | 33 |
| 216 | 47124 | 30536352 |
| 108 | 70686 | 30536352 |
| 1296 | 15708 | 2)33589.9872 |
| | 7854 | |
| | 1017.8784 | 16794.9936 |
| | 1728)16794 99 36(9.71 Feet, the Content: | |
| | 5552 | |
| | 12249 | |
| | 12096 | |
| | 3339 | |
| | 1728 | |
| | 1611 | |

Demonstration. The Parabolic Conoid is constituted of an infinite Number of Circles, whose Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, it will be, $SA : AB :: AB : \frac{AB^2}{SA} = L$, the *Latus Rectum*.

Then



$$\text{Then } \begin{cases} Sa \times L = \square ba, \\ Se \times L = \square fe, \\ Sy \times L = \square gy, \text{ \&c.} \end{cases}$$

Here $Sa \times L$, $Se \times L$, $Sy \times L$, &c. are a Series of Terms in arithmet. Progreff. Therefore $\square ba$, $\square fe$, $\square gy$, &c. are also a Series of Terms in the same Progreffion, beginning at the Point S, wherein $\square AB$ is the greatest Term, and SA , the Number of all the Terms. Therefore $\square AB \times \frac{1}{2} SA =$ the Sum of all the Series. (By Lemma II.)

Consequently, $\odot AB \times \frac{1}{2} SA =$ the Sum of all the Series of $\odot ba$, $\odot fe$, $\odot gy$, &c. which do constitute the Solidity of the Conoid.

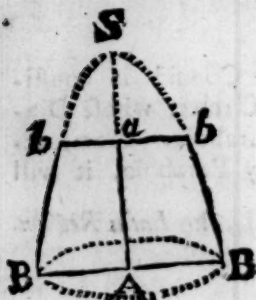
Put $D = 2AB$, and $H = SA$.

Then $.7854 DD \times \frac{1}{2} H = .3927 DDH$ will be the solid Content of the Conoid; which is just half the Cylinder, whose Base is $= D$, and Height $= H$.

This being rightly understood, it will be easy to raise a Theorem for finding the lower Frustum of any Parabolic Conoid.

For, supposing $h = aA$, the Height of the Frustum, and $p = Sa$, the Height of the Part bSb cut off, and $h + p = SA$, the Height of the whole Conoid.

Consequently, $\frac{\odot AB \times H + \odot AB \times p}{2} =$ the Solidity of the whole Conoid.



And $\frac{\odot ba \times p}{2} =$ the Solidity of the Part cut off.

Therefore

| | | | |
|-------------------------|---|-------------------------------|-------------------------------------------------------------------|
| Therefore | 1 | { | $\odot AB \times h + \odot AB \times p - \odot ba \times p$ |
| | | | $\frac{2}{}$ |
| | | the Solidity of the Fruustum. | |
| But | 2 | | $h + p : \square AB :: p : \square ba$ |
| Conseq. | 3 | | $h + p : \odot AB :: p : \odot ba$ |
| 3 .. | 4 | | $\odot AB \times p : \odot ba \times ph + \odot ba \times p$ |
| 4 — $\odot ba \times p$ | 5 | | $\odot AB \times p : \odot ba \times h \odot ba \times h$ |
| | 6 | { | $\odot AB \times h : + \odot AB \times p : - \odot ba$ |
| 1 x 2 | | | $\times p = 2F.$ |
| 6 — 5 | 7 | | $\odot AB \times h = 2F : - \odot ba \times h.$ |
| 7 + $\odot ba \times h$ | 8 | | $\odot AB \times h : + \odot ba \times h = 2F.$ |
| | 9 | { | $\frac{\odot AB + \odot ba}{2} : \times h = F, \text{ the Frus-}$ |
| 8 ÷ 2 | | | tum's Solidity. |

Let $D = 2 AB$, as before, and $d = 2 ba$, the Diameter of the Part cut off; then we shall have the following Theorem.

$0.3927 DD + 0.3927 dd : \times h =$ the Solidity of the Fruustum required: Which in Words is thus:

Multiply the Sum of the Squares of the greater and lesser Diameters by .3927, and the Product by the Height of the Fruustum, the last Product shall be the solid Content.



§ XIV. *Of a Parabolic SPINDLE.*

IF an acute Parabola be supposed to be moved about its greatest Ordinate, it will form a Solid, called a Parabolic Spindle. To find the solid Content, this is

The

The RULE.

Multiply the Square of the Diameter of its greatest Circle by .41888 (being $\frac{8}{5}$ of .7854) and that Product by its Length; that last Product is the solid Content.

Let ABCD be a Parabolic Spindle, whose greatest Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is required,

$$\begin{array}{r}
 36 = CD \quad .41888 \\
 36 \quad 1296 \\
 \hline
 216 \quad 251328 \\
 108 \quad 376992 \\
 \hline
 1296 \text{ Square.} \quad 41888 \\
 \hline
 542.86848 \\
 99 \\
 \hline
 488581632 \\
 488581632 \\
 \hline
 1728) 53743.97952 (31.10184 \\
 \quad \cdot \cdot \cdot \cdot \cdot \\
 \quad \hline
 \quad 1903 \\
 \quad 1759 \\
 \quad 3179 \\
 \quad 14515 \\
 \quad 6912 \\
 \quad \hline
 \quad \cdot \cdot \cdot \cdot
 \end{array}$$



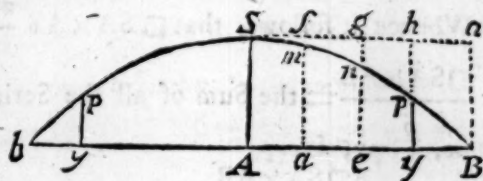
The solid Content is 31.10184 Feet.

Demonstration.

Demonstration. A Parabolic Spindle is constituted of an infinite Series of Circles, whose Diameters are all parallel to the Axis of the Parabola, as $\odot m a$, $\odot n e$, $\odot p y$, &c.

Let us suppose the Line $S d$ parallel to $A B$, &c. Then it hath already been proved, that the Lines $f m$, $g n$, $h p$, &c.

are a Series of Squares, whose Roots are in arithmetical Progression, consequently their Squares, *viz.* $\square f m$, $\square g n$, $\square h p$, &c. will be a Series of Biquadrats, whose Roots will be in arithmetical Progression: Which being premised, we may proceed thus:



First $\left\{ \begin{array}{l} 1 \quad SA - fm = ma. \\ 2 \quad SA - gn = ne. \\ 3 \quad SA - hp = py. \end{array} \right.$

1 \odot 2 $\left\{ \begin{array}{l} 4 \quad \square SA - 2SA \times fm + \square fm = \square ma. \\ 5 \quad \square SA - 2SA \times gn + \square gn = \square ne. \\ 6 \quad \square SA - 2SA \times hp + \square hp = \square py, \text{ \&c.} \end{array} \right.$

1. In these Equations, the $\square SA$, $\square SA$, $\square SA$, being a Series of Equals, and AB the Number of all the Terms; therefore it will be $\square SA \times AB =$ the Sum of the Series. (By Lemma I.)

2. Because fm , gn , hp , &c. are a Series of Squares, wherein SA is the greatest Term, and AB the Number of all the Terms.

Therefore $\frac{2SA \times SA \times AB}{3} = \frac{2 \square SA \times AB}{3}$ will be the Sum of all the Series. (By Lemma III.)

3. And

3. And the \square sm, \square gn, \square hp, &c. will be a Series of Terms in the Ratio of Biquadrats, as above; \square SA being the greatest Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series. (By Lemma V.)

Whence it follows, that $\square SA \times AB \frac{2 \square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of \square ma, \square ne, \square py, &c.

That is, $\frac{8 \square SA \times AB}{15}$ = the Sum of all the Series \square me, \square ne, \square py, &c. Consequently, $\frac{8 \odot SA \times AB}{15}$ = the Sum of all the Series of Circles, \odot ma, \odot ne, \odot py, &c. which constitute the Solidity of half the Spindle; viz. of SAB.

Therefore putting $D = 2SA$, and $H = 2AB$, it will be 0.41888 DDH = the Solidity of the whole Parabolic Spindle bSB, being $\frac{8}{15}$ of 0.7854 DDH, the Solidity of its circumscribing Cylinder.

From hence we may also raise a Theorem for finding the Frustrum, SApy, of the last Figure.

For $\odot SA$ being the greatest Term, $\odot py$ the least Term, and Ay the Number of all the Terms or Circles included between A and y:

Therefore,

There-fore { 1 $\square SA \frac{2 SA \times hp}{3} + \frac{\square hp}{5} : \times Ay =$
 z the Sum of all the Series, $\square SA$, \square
 ma , $\square en$, $\square py$.

1 $\times 3$ 2 $3 \square SA - 2 SA \times hp + \frac{3 \square hp}{5} : \times Ay = 3z$

$z - Ay$ 3 $3 \square SA - 2 SA \times hp + \frac{3 \square hp}{5} = \frac{3z}{Ay}$

But 4 $\square SA - 2 SA \times hp = \square py - \square hp$, per St. 6.

3-4 5 $2 \square SA + \frac{\square 3hp}{5} = \frac{3z}{Ay} - \square py + \square hp$.

5+6c. 6 $2 \square SA + \square py - \frac{2}{3} \square hp = \frac{3z}{Ay}$

Conseq. 7 $2 \square SA + \square py - \frac{2}{3} \square hp : \times \frac{2}{3} Ay = z$
the Sum of all the Series of $\square SA$, $\square ma$, $\square ne$, $\square py$;
which do constitute the Solidity of the Frustum $SApy$.
Therefore putting $D = 2 SA$, as before, $C = 2 py$,
 $x = 2 hp$, and $H = Ay$; it will be $1.5708 DD + .7854 CC - .31416xx : \times \frac{1}{3} H =$ the Frustum $SApy$.
And if we make $L = 2H$, then $1.5708 DD + .7854 CC - .31416 xx : \times \frac{1}{3} L =$ the Double of that Frustum, being the middle Zone. Which in Words is thus:

Multiply the Square of the greatest Diameter by 1.5708, and multiply the Square of the lesser Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products subtract the latter Product, and multiply the Remainder by one third Part of the Length, and that Product will be the Solidity of the middle Zone required.



C H A P. III.

Of the Measuring the Works of the several Artificers relating to Building; and what Methods and Customs are observed therein.

§ I. *Of CARPENTERS Work.*

THE Carpenters Works, which are measurable, are Flooring, Partitioning, and Roofing; all which are measured by the Square of 10 Feet long, and 10 Feet broad; so that one Square contains 100 Square Feet.

1. *Of Flooring.*

. If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad; How many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, &c. which divide by 100 (this is done by cutting from the Product two Figures towards the Right-hand, with a Dash of the Pen); and the remaining Figures are the Quotient, and

and the Figures cut off are Feet: Thus, 1631 divided by 100, by cutting off 31 from the Right-hand thereof, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work both by Decimals, and also by Feet and Inches.

| $ \begin{array}{r} 57.25 \\ 28.5 \\ \hline 28625 \\ 45800 \\ 11450 \\ \hline 16 31.625 \end{array} $ | <table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">F.</th> <th style="text-align: left;">I.</th> </tr> <tr> <td>57</td> <td>3</td> </tr> <tr> <td>28</td> <td>6</td> </tr> <tr> <td colspan="2"><hr/></td> </tr> <tr> <td>456</td> <td></td> </tr> <tr> <td>114</td> <td></td> </tr> <tr> <td>28</td> <td>7 6</td> </tr> <tr> <td>7</td> <td>0 0</td> </tr> <tr> <td colspan="2"><hr/></td> </tr> <tr> <td>16 31</td> <td>7 6</td> </tr> </table> | F. | I. | 57 | 3 | 28 | 6 | <hr/> | | 456 | | 114 | | 28 | 7 6 | 7 | 0 0 | <hr/> | | 16 31 | 7 6 |
|-----------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|----|----|---|----|---|-------|--|-----|--|-----|--|----|-----|---|-----|-------|--|-------|-----|
| F. | I. | | | | | | | | | | | | | | | | | | | | |
| 57 | 3 | | | | | | | | | | | | | | | | | | | | |
| 28 | 6 | | | | | | | | | | | | | | | | | | | | |
| <hr/> | | | | | | | | | | | | | | | | | | | | | |
| 456 | | | | | | | | | | | | | | | | | | | | | |
| 114 | | | | | | | | | | | | | | | | | | | | | |
| 28 | 7 6 | | | | | | | | | | | | | | | | | | | | |
| 7 | 0 0 | | | | | | | | | | | | | | | | | | | | |
| <hr/> | | | | | | | | | | | | | | | | | | | | | |
| 16 31 | 7 6 | | | | | | | | | | | | | | | | | | | | |

Facit 16 Squares and 31 Feet.

Note, That 5 is the Decimal for half of any thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; so in the last Example, .25 is the Decimal of 3 Inches, because 3 Inches is a Quarter of a Foot; and 5 is the Decimal of 6 Inches, because 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad; How many Squares are contained in that Floor?

| $ \begin{array}{r} 47.75 \\ 53.5 \\ \hline 23875 \\ 14325 \\ 23875 \\ \hline 25 54.625 \end{array} $ | <table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">F.</th> <th style="text-align: left;">I.</th> </tr> <tr> <td>53</td> <td>6</td> </tr> <tr> <td>47</td> <td>9</td> </tr> <tr> <td colspan="2"><hr/></td> </tr> <tr> <td>371</td> <td></td> </tr> <tr> <td>212</td> <td></td> </tr> <tr> <td>26</td> <td>9</td> </tr> <tr> <td>13</td> <td>4 6</td> </tr> <tr> <td>23</td> <td>6</td> </tr> <tr> <td colspan="2"><hr/></td> </tr> <tr> <td>25 54</td> <td>7 6</td> </tr> </table> | F. | I. | 53 | 6 | 47 | 9 | <hr/> | | 371 | | 212 | | 26 | 9 | 13 | 4 6 | 23 | 6 | <hr/> | | 25 54 | 7 6 |
|-----------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|----|----|---|----|---|-------|--|-----|--|-----|--|----|---|----|-----|----|---|-------|--|-------|-----|
| F. | I. | | | | | | | | | | | | | | | | | | | | | | |
| 53 | 6 | | | | | | | | | | | | | | | | | | | | | | |
| 47 | 9 | | | | | | | | | | | | | | | | | | | | | | |
| <hr/> | | | | | | | | | | | | | | | | | | | | | | | |
| 371 | | | | | | | | | | | | | | | | | | | | | | | |
| 212 | | | | | | | | | | | | | | | | | | | | | | | |
| 26 | 9 | | | | | | | | | | | | | | | | | | | | | | |
| 13 | 4 6 | | | | | | | | | | | | | | | | | | | | | | |
| 23 | 6 | | | | | | | | | | | | | | | | | | | | | | |
| <hr/> | | | | | | | | | | | | | | | | | | | | | | | |
| 25 54 | 7 6 | | | | | | | | | | | | | | | | | | | | | | |

Facit 25 Squares and 54 Feet.

By Scale and Compasses.

In the first Example, extend the Compasses from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the second Example, extend the Compasses from 1 to 47.5, that Extent will reach from 53.5 to 25 Squares and above a Half.

2. Of Partitioning.

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1010.625; which divided by 100 (as before is shewed) and the Answer is 10 Squares 10 Feet; the Inches or Parts, in these Cases, are of no Value.

| | |
|-----------|-----------|
| 12.25 | F. I. |
| 82.5 | 82 6 |
| <hr/> | 12 3 |
| 6125 | <hr/> |
| 2450 | 990 0 |
| 9800 | 20 7 6 |
| <hr/> | <hr/> |
| 10 10.625 | 10 10 7 6 |

Facit 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and its Breadth 11 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1032 Feet; which divided by 100, the Answer will be 10 Squares and 32 Feet.

| | |
|------------|-----------|
| 91.75 | F. I. |
| 11.25 | 61 9 |
| <hr/> | 11 3 |
| 45875 | <hr/> |
| 18350 | 1009 3 |
| 9175 | 22 11 3 |
| 9175 | <hr/> |
| 10 32.1875 | 10 32 2 3 |

3. *Of Roofing.*

It is a Rule amongst Workmen, that the Flat of any House, and half the Flat thereof, taken within the Walls, is equal to the Measure of the Roof of the same House; but this is when the Roof is true pitched: For if the Roof be more flat or steep than the true Pitch, it will measure to more or less accordingly.

Example 1. If a House within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; How many Squares of Roofing will cover that House?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet; which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Answer is 12 Squares and 18 Feet.

| | | | |
|------|---------|----------|---------|
| | 18.25 | F. | I. |
| | 44.5 | 44 | 6 |
| | <hr/> | 18 | 3 |
| | 9125 | <hr/> | |
| | 7300 | 352 | |
| | 7300 | 44 | |
| | <hr/> | 11 | 1 6 |
| Flat | 812.125 | 9 | 0 0 |
| Half | 406 | <hr/> | |
| | <hr/> | The Flat | 812 1 6 |
| | 12 18 | The Half | 406 |
| | | <hr/> | |
| | | Sum | 12 18 |

Facit 12 Squares 18 Feet.

By Scale and Compasses.

In the first Example of Partitioning, extend the Compasses from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and One Tenth.

In the second Example, extend the Compasses from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little less than a third Part.

In the Example of Roofing extend the Compasses from 1 to 18.25, that Extent will reach from 44.5 to 812, the Flat; to which add the Half thereof, and the Sum is 12:18; which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, which are measured by the Foot, running Measure, that is, by the Number of Feet in Length only; as Cornices, Doors and Cases, Window-frames, Guttering, Lintels, Sommers, Skirt-boards, &c.

Note 1. In the Measuring of Flooring, after you have measured the whole Floor, you must deduct out of it the Well-holes for the Stairs and Chimnies; and in Partitioning, for the Doors, Windows, &c. except (by Agreement) they are to be included.

Note 2. In measuring of Roofing, seldom any Reductions are made for the Holes for the Chimney-shafts, the Vacancies for Lutheren-lights and Sky-lights; for they are more Trouble to the Workman than the Stuff which would cover them is worth.



§ II. Of BRICKLAYERS *Work.*

THE principal is Tiling, Walling, and Chimney-work.

1. Of Tiling.

Tiling is measured by the Square of 100 Feet, as Flooring, Partitioning, and Roofing were in the Carpenters Work; so that between the Roofing and Tiling, the Difference will not be much; yet the Tiling will be the most; for the Bricklayers sometimes will require to have double Measure for Hips and Vallies. When Gutters are allowed double Measure, the Way is to measure the Length along the Ridge-tile, and by that means the Measure of the Gutters becomes double; it is usual also to allow double Measure at the Eaves, so much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

| | | |
|-------|------|----------|
| F. | I. | |
| 37 | 3 | 37.25 |
| 45 | 0 | 45 |
| <hr/> | | <hr/> |
| 185 | | 18625 |
| 148 | | 14900 |
| <hr/> | | <hr/> |
| 11 | 3 | 16 762.5 |
| <hr/> | | |
| 16 | 76 3 | |

Answer, 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whose Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tiling are in the Roof?

| | | |
|-------|--------|-----------|
| F. | I. | |
| 43 | 6 | 35.75 |
| 35 | 9 | 43.5 |
| <hr/> | | <hr/> |
| 215 | | 17875 |
| 129 | | 10725 |
| 21 | 9 | 14300 |
| 10 | 10 6 | <hr/> |
| 17 | 6 | |
| <hr/> | | 15 55.125 |
| 15 | 55 1 6 | |

Here the Length and Depth being multiplied together, the Product is 1555 Feet; which divided by 100 (as before is taught) the Answer is 15 Squares and 55 Feet.

By Scale and Compasses.

In the first Example, extend the Compasses from 1 to 37.25, that Extent will reach from 45 to 16 Squares and a little above three Quarters of a Square.

In

Chap. 3. BRICKLAYERS *Work.* 213

In the second Example, extend the Compasses from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 35 Feet; that is, a little above a Half-square.

2. *Of Walling.*

Bricklayers commonly measure their Work by the Rod Square of 16 Feet and a half; so that one Rod in Length, and one in Breadth, contain 272.25 Square Feet; for 16.5, multiplied in itself, produces 272.25 Square Feet. But in some Places the Custom is to allow 18 Feet to the Rod; that is, 324 Square Feet. And in some Places the usual Way is, to measure by the Rod of 21 Feet long and 3 Feet high, that is, 63 Square Feet; and here they never regard the Thickness of the Wall, but the usual Way is to moderate the Price according to the Thickness.

When you measure a Piece of Brick-work, the first thing is to enquire by which of those Ways it must be measured; then, having multiplied the Length and Breadth in Feet together, divide the Product by the proper Divisor, either for Rods or Roods, and the Quotient is Square Rods, or Square Roods, accordingly.

But commonly Brick-walls, that are measured by the Rod, are to be reduced to a Standard-thickness; *viz.* of a Brick and a half thick (if it be not agreed on the contrary); and to reduce a Wall to Standard-thickness, this is

The R U L E.

Multiply the Number of superficial Feet that are found to be contained in any Wall by the Number of Half-bricks which that Wall is in Thickness; one third Part of that Product shall be the Content thereof in Feet, reduced to the Standard-thickness of one Brick and a half.

Example

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick; How many Rods of Brick-work are contained therein, when reduced to the Standard?

19.25 Height.

72.5 Length.

9625
3850
13475

1395.625
11

3)15351.875

272.25) 5117.291 (18 Rods.

239479

68.06)21675 (3 Quarters of a Rod.

12.61

Answer, 18 Rods 3 Quarters 12 Feet.

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$$\begin{array}{r} \text{F. I.} \\ 72 \quad 6 \\ 19 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 648 \\ 72 \\ 18 \quad 1 \quad 6 \\ 9 \quad 6 \quad 0 \\ \hline \end{array}$$

11

$$\begin{array}{r} 3 \overline{)13345} \\ \hline \end{array}$$

$$272 \overline{)5115} (18 \text{ Rods.}$$

$$\begin{array}{r} 2395 \\ \hline \end{array}$$

$$68 \overline{)219} (3 \text{ Quarters of a Rod.}$$

$$\begin{array}{r} 15 \\ \hline \end{array}$$

Note, That 68.06 is one-fourth Part of 272.25.

Note also, That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as is done in the second Way of the last Example; so the Answer, by the second Way, is 18 Rods 3 Quarters and 15 Feet; more by about $2\frac{1}{2}$ Feet than by the first Way, where it is done decimally; a thing very insignificant.

Example 2. If a Wall be 245 Feet 9 Inches long, 16 Feet 6 Inches high, and two Bricks and a half thick; I demand how many Rods of Brick-work are contained therein, when reduced to Standard-thickness?

$$\begin{array}{r}
 245.75 \\
 16.5 \\
 \hline
 122875 \\
 147450 \\
 24575 \\
 \hline
 4054.875 \\
 5 \\
 \hline
 3)20270 \\
 \hline
 272)6756(24 \text{ Rods.} \\
 \hline
 1316 \\
 \hline
 68)228(3 \text{ Quarters of a Rod.} \\
 \hline
 24
 \end{array}$$

Answer, 24 Rods 3 Quarters 24 Feet.

F. I.

245 9
16 6

1470

245

122 10 6

12 0 0

4054 10 6 Answer in Feet.

Before I shew how to work the two last Examples by Scale and Compasses, I will shew how to find proper Divisors to facilitate the Operation; because it would be too intricate and tedious to perform by Scale and Compasses, according to the Rule above taught.

To find proper Divisors.

Divide 3 (the Number of Half-bricks in $1\frac{1}{2}$), by the Number of Half-bricks in the Thickness, the Quotient will be a Divisor, which will give the Answer in Feet. But if you would have a Divisor to bring the Answer in Rods at once, then multiply 272.25 by the Divisor found for Feet, and the Product will be a Divisor, which will give the Answer in Rods.

Example. Let it be required to find a Divisor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6 (the Half-bricks in the Thickness) and the Quotient is .5, which is a Divisor that will give the Answer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Divisor, which will give the Answer in Rods; that is, as 136.125 is to the Length of the Wall, so is the Height to the Content in Rods. Or, as .5 is to the Length, so is the Height to the Content in Feet.

After the same Manner you may find Divisors for any other Thickness, which you will find to be as expressed in the following little Table.

| The Thickness of the Wall. | Divisors for the Answer in Feet. | Divisors for bringing the Answer in Rods. |
|-------------------------------|-------------------------------------------|----------------------------------------------------|
| 1 Brick thick | 1.5 | 408.375 |
| 1½ Brick thick | 1. | 272.25 |
| 2 Bricks thick | .75 | 204.1875 |
| 2½ Bricks thick | .6 | 163.35 |
| 3 Bricks thick | .5 | 136.125 |
| 3½ Bricks thick | .4285 | 116.659 |
| 4 Bricks thick | .375 | 102.0937 |

Let the second Example, foregoing, be wrought by Scale and Compasses, where the Length is 245.75, the Height 16.5, and the Thickness $2\frac{1}{2}$ Bricks.

Extend the Compasses from 163.35 (the tabular Number against $2\frac{1}{2}$ Bricks), to 245.75; that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at $3\frac{1}{2}$ Bricks thick; How many Rods are contained therein?

Extend the Compasses from 116.659 (the tabular Number) to 18.75, that Extent will reach from 75 5 to 12.13, that is, 12 Rods and a little above half a Quarter.

It will be very proper and commodious, for such as have frequent Occasion to measure Brick-work, to have in the Line of Numbers little Brass Center-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickness of the Wall.

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If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high; How many Rods are contained therein?

| $ \begin{array}{r} 104.75 \\ 17.25 \\ \hline 52375 \\ 20950 \\ 73325 \\ 10475 \\ \hline 631806.9375(28 \\ 126 \\ \hline \end{array} $ | <table style="width: 100%; border-collapse: collapse;"> <tr> <th style="text-align: left;">F.</th> <th style="text-align: left;">I.</th> </tr> <tr> <td>104</td> <td>2</td> </tr> <tr> <td>17</td> <td>3</td> </tr> <tr> <td colspan="2"><hr/></td> </tr> <tr> <td>728</td> <td></td> </tr> <tr> <td>104</td> <td></td> </tr> <tr> <td>26</td> <td>2 3</td> </tr> <tr> <td>12</td> <td>9 0</td> </tr> <tr> <td colspan="2"><hr/></td> </tr> <tr> <td>1806</td> <td>11 3</td> </tr> </table> | F. | I. | 104 | 2 | 17 | 3 | <hr/> | | 728 | | 104 | | 26 | 2 3 | 12 | 9 0 | <hr/> | | 1806 | 11 3 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|----|-----|---|----|---|-------|--|-----|--|-----|--|----|-----|----|-----|-------|--|------|------|
| F. | I. | | | | | | | | | | | | | | | | | | | | |
| 104 | 2 | | | | | | | | | | | | | | | | | | | | |
| 17 | 3 | | | | | | | | | | | | | | | | | | | | |
| <hr/> | | | | | | | | | | | | | | | | | | | | | |
| 728 | | | | | | | | | | | | | | | | | | | | | |
| 104 | | | | | | | | | | | | | | | | | | | | | |
| 26 | 2 3 | | | | | | | | | | | | | | | | | | | | |
| 12 | 9 0 | | | | | | | | | | | | | | | | | | | | |
| <hr/> | | | | | | | | | | | | | | | | | | | | | |
| 1806 | 11 3 | | | | | | | | | | | | | | | | | | | | |

Answer, 28 Rods 42 Feet.

$$\begin{array}{r}
 546 \\
 504 \\
 \hline
 42
 \end{array}$$

Note, That such as dig Cellars, do many times make them by the Floor, 16 Feet square, and a Foot deep, being a Floor of Earth; that is, 324 solid Feet.

3. Of Chimnies.

If you are to measure a Chimney standing alone by itself, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickness must be the same as the Jambs are of, provided that the Chimney be wrought upright from the Mantle-tree to the Ceiling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, because of the Gatherings of the Breast and Wings, to make room for the Hearth in the next Story.

If the Chimney-back be a Party-wall, and the Wall be measured by itself, then you must measure the Depth of the two Jambs, and the Length of the Breast for a Length, and the Height of the Story the Breadth, at the same Thickness your Jambs were of.

When you measure Chimney-shafts, girt them with a Line round about the least Place of them, for

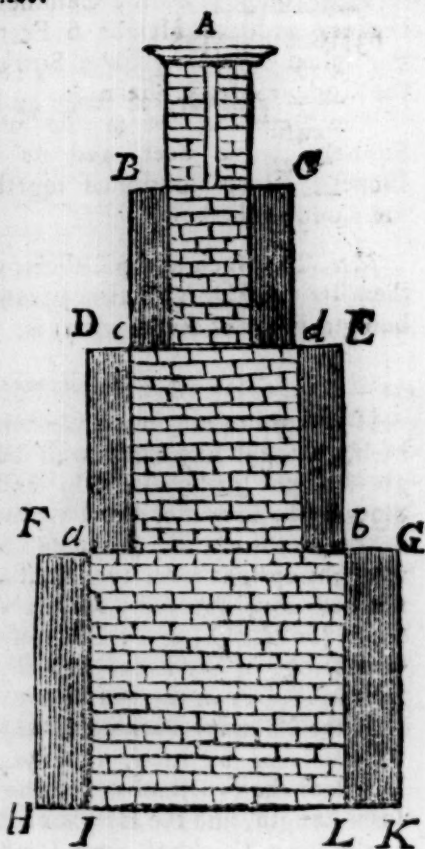
U 2 the

the Length, and the Height shall be your Breadth : And if they be four Inch-work, then you must set down their Thickness at one Brick-work ; but if they be wrought 9 Inches thick (as sometimes they are, when they stand high and alone above the Roof), then you must account your Thickness $1\frac{1}{2}$ Brick, in Consideration of Widths and Pargetting, and Trouble in Scaffolding.

It is customary, in most Places, to allow double Measure for Chimnies.

Example. Suppose this Figure, ABCDEFGHIK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be measured according to double Measure.

First, I begin with the Breast-wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches ; then take the Height of the Square HF, 12 Feet 6 Inches ; which multiplied together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.



For

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For the Square DaEb, the Length of the Breast-wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiplied together, make 130 Feet 6 Inches, for the Content of the Part DaEb.

Then the Height of the next Square 7 Feet, and the Length of the Breast-wall and two Angles is 10 Feet 3 Inches; which multiplied together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compass of the Chimney-shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiplied together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideness 1 Foot 3 Inches; which multiplied together, make 15 Feet, the Content thereof.

The Work:

| F. | I. | |
|-------|-----|-------|
| 18 | 9 | 18.75 |
| 12 | 6 | 12.5 |
| <hr/> | | |
| 225 | 0 | 9375 |
| 9 | 4 6 | 3750 |
| <hr/> | | 1875 |

FGHK 234 4 6

FGHK 234.375

| F. | I. | |
|-------|----|------|
| 14 | 6 | 14.5 |
| 9 | 0 | 9 |
| <hr/> | | |

DaEb 130 6

DaEb 130.5

| F. | I. | |
|-------|----|-------|
| 10 | 3 | 10.25 |
| 7 | 0 | 7 |
| <hr/> | | |

BcCd 71 9

BcCd 71.75

| | |
|------------------|------------------|
| F. I. | 13.73 |
| 13 9 | 6.5 |
| 6 6 | |
| <hr/> | |
| 82 6 | 6875 |
| 6 10 6 | 8250 |
| <hr/> | |
| The Shaft 89 4 6 | The Shaft 89.375 |

| | |
|-----------------|------------------|
| F. I. | 1.25 |
| 1 3 | 12 |
| 12 0 | |
| <hr/> | |
| The Fetter 15 0 | The Fetter 15.00 |

| | | | |
|---------------------|-----------------|----|----|
| | F. | I. | P. |
| 272)1082(3 Rods. | FGHK 234 | 4 | 6 |
| | DaEb 130 | 6 | 0 |
| | BcCd 71 | 9 | 0 |
| 68) 266(3 Quarters. | The Shaft 89 | 4 | 6 |
| | The Fetter 15 | 0 | 0 |
| Rem. 62 Feet. | | | |
| | The Sum 541 | 0 | 0 |
| | | | |
| | The Double 1082 | 0 | 0 |

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or customary Measure; which Feet must be reduced to Rods, as was shewed before.

So the Feet in the foregoing Example being reduced to Rods (the Thickness being supposed $1\frac{1}{2}$ Brick) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all the Measure that can be allowed, when the Chimney stands in a Gavel or Side-wall; in which Case the Back of the Chimney (here not measured) is accounted as Part of the Gavel; but if the Chimnies stand by themselves, as all Stacks of Chimnies

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in great Buildings do, in such Case, it is all Chimney-work, and therefore ought to be measured double on all Sides.



§ III. Of PLASTERERS Work.

THE Plasterers Works are principally of two Kinds; namely, 1. Works lathed or plastered, which they call Cieling. 2. Works rendered; which are of two Kinds; viz. upon Brick walls, or between Quarters, in the Partitions between Rooms: All which are measured by the Yard-square, or Square, of 3 Feet, which is 9 Feet.

1. Of Cieling.

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad; How many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

| | |
|-----------|---------------|
| F. I. | 59.75 |
| 59 9 | 24 5 |
| 24 6 | <hr/> |
| <hr/> | 29875 |
| 236 | 23900 |
| 118 | 11950 |
| 29 10 6 | <hr/> |
| 18 0 0 | 9)1463.875 |
| <hr/> | <hr/> |
| 1463 10 6 | Answer 162.65 |

By Scale and Compasses.

Extend the Compasses from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 16.25 Yards.

2. Of Rendering.

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high; How many Yards are in those Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Answer.

| F. I. | |
|--------------|------------|
| 141 6 | 141.5 |
| 11 3 | 11.25 |
| 1556 6 | 7075 |
| 35 4 6 | 2830 |
| 9)1591 10 6 | 1415 |
| Answer 176 7 | 1415 |
| | 9)1591.875 |
| | 176.87 |

Answer, 176.87 Yards.

Extend the Compasses from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning, you must make Deductions for them.

Note 2. When you measure Rendering upon Brick Walls, you are to make no Deductions; but when you measure Rendering between Quarters, you may very well deduct one fifth Part for the Quarters, Braces, and Interstices.

Note.

Note 3. That Whiting and Colouring are both measured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deduct one fifth Part, so in Whiting and Colouring you must add one fourth or one fifth Part at least.



§ IV. Of JOYNEERS Work.

JOYNEERS measure their Work by the Yard-square; but in taking their Dimensions, they differ from some others; for they have a Custom, and say, *We ought to measure where our Plane touches*: Wherefore in taking the Height of any Room, where there is a Cornice about, and swelling Panels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to measure much higher than it is: Then for measuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room or Wainscot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, add 126 Feet and 3 Inches in Compass; How many Yards doth that Room contain?

Multiply the Compass by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Answer.

| F. | I. | |
|---------------|-----|-------------|
| 126 | 3 | 126.25 |
| 15 | 9 | 15.75 |
| <hr/> | | <hr/> |
| 620 | | 63125 |
| 126 | | 88375 |
| 63 | 1 6 | 63125 |
| 31 | 6 9 | 12625 |
| 3 | 9 0 | <hr/> |
| 9)1988 | 3 3 | 9)1988.4375 |
| <hr/> | | <hr/> |
| Answer, 220 8 | | 220.8 |

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainscot be 16 Feet 3 Inches high (being girt over the Mouldings), and the Compass of the Room 137 Feet 6 Inches; How many Yards are contained therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

| F. | I. | |
|--------|-----|------------|
| 137 | 6 | 137.5 |
| 16 | 3 | 16.25 |
| <hr/> | | <hr/> |
| 830 | | 6875 |
| 137 | | 2750 |
| 34 | 4 6 | 8250 |
| 9)2234 | 4 6 | 1375 |
| <hr/> | | <hr/> |
| 248 | 2 0 | 9)2234 375 |
| | | <hr/> |
| | | 248.2 |

Facit 248 Yards 2 Feet.

By Scale and Compasses.

For the first Example, extend the Compasses from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the second Example, extend the Compasses from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards and above a Quarter.

In Joyner's Work there is another thing to be observed; that is, in the measuring of Doors, Window-shutters, and all such Work as is wrought on both Sides, they are paid for Work and Half work; so that in measuring all such Work, you must first find the Content, as before, and take half that Content, and add to it; so shall the Sum be the Content at Work and half.

Example. If the Window shutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high; How many Yards are contained therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the Half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

| F. I. | |
|-------------------------|----------|
| 69 9 | 69 75 |
| 6 3 | 6.25 |
| 418 6 | 34875 |
| 17 5 3 | 13950 |
| 435 11 3 | 41850 |
| 217 11 7 | 435 9375 |
| 9)653 10 10 | 217 9687 |
| 27 5 | 653.9062 |
| Facit 248 Yards 5 Feet. | |

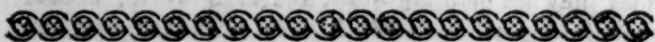
By

By

By Scale and Compasses.

Extend the Compasses from 9 to 69 75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.02; which added together, make 72.6 Yards, the Content at Work and half.

Note, That you must make Deductions for all Window-lights; but you must measure the Window-boards, Sopheta boards, and Cheeks, by themselves.



§ V. Of PAINTERS Work.

THE taking the Dimensions of Painters Work is the same as that of Joyners, by girting over the Mouldings and swelling Panels, in taking the Height; and it is but Reason that they should be paid for that on which their Time and Colour are both expended. The Dimensions thus taken, the casting up, and reducing Feet into Yards, is altogether the same as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice coloured over. Only take Notice, that Window-lights, Window-bars, Casements, and such-like Things, they do at so much *per Piece*.

Example. If a Room be painted, whose Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compass of the Room 97 Feet 9 Inches; How many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 0 Inches 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.

| F. | I. | |
|--------|------|----------------|
| 97 | 9 | 97.75 |
| 16 | 6 | 16.5 |
| <hr/> | | <hr/> |
| 584 | | 48375 |
| 98 | | 58650 |
| 48 | 10 6 | 9775 |
| <hr/> | | <hr/> |
| 9)1612 | 10 6 | 9)1612.875(179 |
| <hr/> | | |
| 179 | 1 | |

Facit 179 Yards 1 Foot.

By Scale and Compaffes.

Extend the Compaffes from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.



§ IV. Of GLASIERS Work.

GLASIERS measure their Work by the Foot square; fo that the Length and Breadth of a Pane of Glafs in Feet, being multiplied into each other, produceth the Content.

Note, That Glafiers usually take their Dimen- fions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part subdivided into 12, &c.

Example. If a Pane of Glafs be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad; How many Feet of Glafs are in that Pane?

The Decimal of $\left\{ \begin{array}{l} 8 \text{ Inches } \frac{3}{4} \\ 4 \text{ Inches } \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .728 \\ .354 \end{array} \right\}$

X

F.

| F. | I. | P. | |
|-------|----|----|---|
| 4 | 8 | 9 | |
| 1 | 4 | 3 | |
| <hr/> | | | |
| 4 | 8 | 9 | |
| 1 | 6 | 11 | 0 |
| | 1 | 2 | 2 |
| | | 3 | |
| <hr/> | | | |
| 6 | 4 | 10 | 2 |
| | | 2 | 3 |

4.729
 1.354

 18916
 23645
 14187
 4729

 6.403066

Answer, 6 Feet 4 Inches.

By Scale and Compasses.

Extend the Compasses from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glafs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad; How many Feet of Glafs are contained in the said 8 Panes?

The Decimal of $\left\{ \begin{array}{l} 7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .646 \\ .437 \end{array} \right\}$

| F. | I. | P. | |
|-------|----|----|----|
| 4 | 7 | 9 | |
| 1 | 5 | 3 | |
| <hr/> | | | |
| 4 | 7 | 9 | |
| 1 | 11 | 2 | 9 |
| | 1 | 1 | 11 |
| | | 3 | |
| <hr/> | | | |
| 6 | 8 | 1 | 8 |
| | | 3 | 8 |
| <hr/> | | | |
| 53 | 5 | 1 | 6 |
| | | 0 | |

4.646
 1.437

 32522
 13938
 18584
 4646

 6.676302
 8

 53.410416

Facit 53 Feet 5 Inches.

By

By Scale and Compasses.

Extend the Compasses from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compasses from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

Example 3. If there be 16 Panes of Glass, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad; How many Feet of Glass are contained therein?

| F. | I. | P. | | |
|-------|----|----|---|-----------|
| 4 | 5 | 6 | | 4.458 |
| 1 | 4 | 9 | | 1.395 |
| <hr/> | | | | <hr/> |
| 4 | 5 | 6 | | 22290 |
| 1 | 5 | 10 | 0 | 40122 |
| | 3 | 4 | 1 | 13374 |
| | | | 6 | 4458 |
| <hr/> | | | | <hr/> |
| 6 | 2 | 8 | 1 | 6.218910 |
| | | | 4 | 4 |
| <hr/> | | | | <hr/> |
| 24 | 10 | 8 | 6 | 24.875640 |
| | | | 4 | 4 |
| <hr/> | | | | <hr/> |
| 96 | 6 | 10 | 0 | 99.502560 |

Facit 99 Feet 6 Inches.

Note, That instead of multiplying by 16, I have multiplied by 4 twice, because 4 times 4 is 16.

By Scale and Compasses.

Extend the Compasses from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compasses from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

Note, That when Windows have Half rounds at the Top, they measure them at the full Height, as if they were square. Also round or oval Windows are measured

X 2.

ured

sured at the full Length and Breadth of their Diameters. Likewise Crotchet-windows in Stone-work are all measured by their full Squares. And there is Reason for so doing; for the Trouble in taking their Dimensions to work by, the Waste of Glafs in working, and the Time expended in setting up, is far more than the Glafs can be valued at.

§ V. Of MASONS Work.

MASONS measure their Work sometimes by the Foot solid, sometimes by the Foot superficial, and in some Places they measure their Walling by the Rood, that is, 21 Feet long and 3 Feet high, which is 63 square Feet. Examples of each are as follow.

Example. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick; How many solid Feet are contained in that Wall?

| F. | I. | |
|-------|-------|--------------|
| 97 | 5 | 97.417 |
| 18 | 3 | 18.25 |
| <hr/> | | <hr/> |
| 776 | | 487085 |
| 97 | | 194834 |
| 24 | 4 3 | 779336 |
| 6 | 0 0 | 97417 |
| 1 | 6 0 | <hr/> |
| <hr/> | | 1777.86025 |
| 1777 | 10 3 | 2.25 |
| 2 | 3 | <hr/> |
| <hr/> | | 888930125 |
| 3555 | 8 6 | 355572050 |
| 444 | 5 6 9 | 355572050 |
| <hr/> | | <hr/> |
| 4000 | 2 0 9 | 4000.1855625 |
| | | Multi. |

Multiply the Length, Height, and Thickness together, and the last Product is 4000 Feet 2 Inches, the solid Feet contained in the Wall.

By Scale and Compasses.

Extend the Compasses from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the solid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high; How many Feet superficial are contained therein?

| F. | I. | |
|-------|------|----------|
| 107 | 9 | 107 75 |
| 20 | 6 | 20 5 |
| <hr/> | | <hr/> |
| 2155 | 0 | 53875 |
| 53 | 10 6 | 215500 |
| <hr/> | | <hr/> |
| 2208 | 10 6 | 2208.875 |

Facit 2208 Feet 10 Inches.

By Scale and Compasses.

Extend the Compasses from 1 to 107.75, that Extent will reach from 20.6 to 2208.875, the superficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high; How many Roods are contained therein?

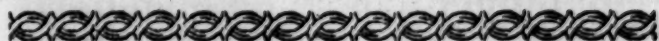
| | |
|------|-----|
| F. | I. |
| 112 | 3 |
| 16 | 6 |
| | |
| 676 | 0 |
| 112 | |
| 56 | 1 6 |
| | |
| 1852 | 1 6 |

| |
|----------------|
| 112.25 |
| 16.5 |
| |
| 56125 |
| 67350 |
| 11225 |
| |
| 63)1852.125(29 |
| |
| 592 |
| |
| 25 |

Facit 29 Roods 25 Feet.

By Scale and Compasses.

Extend the Compasses from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.



C H A P. IV.

The Measuring of BOARD and TIMBER.§ I. *Of BOARD MEASURE.*

TO measure a Board, is no other but to measure a long Square.

Example 1. If a Board be 16 Inches broad and 13 Feet long; How many Feet are contained therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the same as before.

$$12 : 13 :: 16$$

$$\begin{array}{r} 13 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ \hline \end{array}$$

$$12 \overline{) 208}$$

$$17\frac{4}{12}$$

$$\text{Or, } 144 : 156 :: 16$$

$$\begin{array}{r} 16 \\ \hline \end{array}$$

$$\begin{array}{r} 936 \\ \hline \end{array}$$

$$\begin{array}{r} 156 \\ \hline \end{array}$$

$$144 \overline{) 2496} (17\frac{4}{144}$$

$$\begin{array}{r} 1056 \\ \hline \end{array}$$

$$48$$

By Scale and Compasses.

Extend the Compasses from 12 to 13, that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Or, extend from 144 to 156 (the Length in Inches) that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad; How many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and so many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch. Inch. Inch.

$$19 : 144 :: 1 : 7.58 \text{ feet.}$$

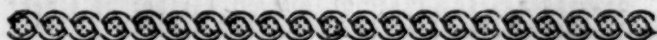
Extend the Compasses from 19 to 144, that Extent will reach from 1 to 7.58; that is, 7 Inches, and something more than a half. So, if a Board be 19 Inches broad, if you take 7 Inches, and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains; or you may cut off from that Board any Number of Feet desired.

For this Purpose there is a Line upon most ordinary Joint rules, with a little Table placed upon the End of all such Numbers as exceed the Length of the Rule, as in this little Table annexed.

| | | | | | | | |
|----|---|---|---|---|---|----|---|
| 0 | 0 | 0 | 0 | 5 | 0 | 8½ | 6 |
| 12 | 6 | 4 | 3 | 2 | 2 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Here you see, if the Breadth be one Inch, the Length must be 12 Feet; if two Inches, the Length is 6 Feet; if five Inches broad, the Length is 2 Feet 5 Inches, &c.

The rest of the Lengths are expressed in the Line, thus: If the Breadth be 9 Inches, you will find it against 15 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, &c.



§ II. Of SQUARED TIMBER.

BY Squared Timber are here meant such as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids are shewed in § II. of Chap. II. to which I refer.

Example

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) square, and 18 Feet long; How many solid Feet are contained therein?

$$\begin{array}{r}
 15 \\
 15 \\
 \hline
 75 \\
 15 \\
 \hline
 225 \\
 18 \\
 \hline
 1800 \\
 225 \\
 \hline
 144 \overline{)4050} (28.125 \\
 \hline
 1170 \\
 \hline
 180 \\
 360 \\
 \hline
 720
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 1 \ 3 \\
 1 \ 3 \\
 \hline
 1 \ 3 \\
 3 \ 9 \\
 \hline
 1 \ 6 \ 9 \\
 6 \\
 \hline
 9 \ 4 \ 6 \\
 3 \\
 \hline
 28 \ 1 \ 6
 \end{array}$$

Answer, 28 Feet and half a Quarter.

Here, instead of multiplying by 18, where I wrought by Feet and Inches, I multiplied by 6, and then by 3, because 3 times 6 is 18.

Example 2. If a Piece of squared Timber be 2 Feet 9 Inches deep; 1 Foot 7 Inches broad, and 16 Feet 9 Inches long; How many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together; and the Product will be the Content.

| | |
|--------------------|-----------|
| 33 | F. I. |
| 19 | 2 9 |
| <hr/> | 1 7 |
| 297 | <hr/> |
| 33 | 2 9 |
| <hr/> | 1 7 3 |
| 627 | <hr/> |
| 16.75 | 4 4 3 |
| <hr/> | 16 8 0 |
| 3135 | <hr/> |
| 4389 | 69 9 0 |
| 3762 | 3 3 2 3 |
| 627 | <hr/> |
| <hr/> | 72 11 2 3 |
| 144)10502.25(72.93 | |
| <hr/> | |
| 422 | |
| 1342 | |
| 465 | |
| <hr/> | |
| 93 | |

Answer, 72 Feet 11 Inches; or 72 Feet 93 Parts.

By Scale and Compasses.

For the first Example, extend the Compasses from 12 to 15 Inches (the Side of the Square, that Extent will reach from 18 Feet (the Length being twice turned over) to 28 Feet, and something more.

For the second Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts, and the Compass Point will rest upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compasses from 12 to 25 (the Proportional found) that Extent will reach (being twice turned over) from 16.75 Feet, the Length, to 7293 Feet, the Content.

A com-

A common Error is committed, for want of Art, in measuring these last Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but small, when the Depth and Breadth are pretty near equal; yet if the Difference be great, the Error is very considerable; for the Piece of Timber thus measured, will be more than the Truth, by a Piece whose Length is equal to the Length of the Piece of Timber to be measured, and the Square equal to half the Difference of the Breadth and Depth, as I shall here demonstrate.

I say, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Square is greater than the Parallelogram by the little Square OHPL; which was to be proved.

Otherwise, you may prove it by Numbers, thus: The Sum of 33 and 19 is 52, the half thereof is 26, the Square of 26 is 676, and the Product of the Depth and Breadth is 627: The Difference of these two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the half thereof is 7, whose Square is 49; which was to be proved.

Now, if this 49 be multiplied by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and those Feet added to the true Content, the

Sum



Chap. 4. *Squared Timber.* 248

Sum will be equal to the Content found by the false Way mentioned.

See the Work of both.

| | |
|--------------------------|-----------------------------------|
| 33 Depth. | 16.75 the Length. |
| 19 Breadth. | 49 the Sq. of $\frac{1}{2}$ Diff. |
| <hr/> 52 Sum. | <hr/> 15075 |
| | 6700 |
| 26 half. | |
| 26 | 144)820.75(5.69 |
| <hr/> 156 | <hr/> 1007 |
| 52 | 1435 |
| <hr/> 676 | <hr/> 139 |
| 16.75 | |
| <hr/> 3380 | |
| 4732 | |
| 4056 | |
| 676 | |
| <hr/> 144)11323.00(78.63 | |
| 1243 | |
| 910 | |
| 460 | |
| <hr/> 28 | |

Feet.

To 72.93 the true Content.
Add 5.69 the Part superfluous

Rem. 78.62 equal to the Content by the false Way.

Y

By

F. I.

0 7

0 7

0 4 1

16 9

5 5 4

3 0 9

5 8 4 9

72 11 2 3

78 7 7 0

Part superfluous.

True Content add.

False C. 78 7 7

equal to the Content by the false Way.

F. I.

2 2

2 2

4 4

4 4

4 8 4

16 9

75 1 4

3 6 3

To find how much in Length makes a Foot of any squared Timber.

Always divide 1728 (the solid Inches in a Foot) by the Area of the Base; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickness from End to End, whether it be square, triangular, multangular, or round.

Example 1. If a Piece of Timber be 18 Inches square; How much in Length will make a Foot solid?

18

18

144

18

324)1728($5\frac{1}{3}$ Inches; which is the Answer.

1620

108

By

By Scale and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 18 to 323, the Square or Area of the Base; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches, and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compasses from 18 to 41.569, that Extent, turned twice over from 1, will at last fall upon $5\frac{1}{3}$, as before.

Note, That 41.569 is the square Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad; How much in Length will make a Foot?

$$\begin{array}{r}
 22 \\
 15 \\
 \hline
 110 \\
 22 \\
 \hline
 330 \overline{)1728(5.23} \\
 \underline{780} \\
 1200 \\
 \underline{210}
 \end{array}$$

Answer, 5 Inches and .23 Parts.

By Scale and Compasses.

Extend the Compasses from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpose upon most ordinary Rules, with a little Table at the End of all such Numbers as exceed the Length of the Rule, such as this annexed.

| | | | | | | | | | |
|-----|----|----|---|---|---|----|---|---|-----------------|
| 0 | 0 | 0 | 0 | 9 | 0 | 11 | 3 | 9 | Inches. |
| 144 | 36 | 16 | 9 | 5 | 4 | 2 | 2 | 1 | Feet. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Side of the Sq. |

Here you see, if the Side of the Square be 1, the Length must be 144 Feet; if two Inches be the Side of the Square, it must be 36 Feet in Length, to make a solid Foot, &c.

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it against 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compasses, and run the Compasses along the Piece from End to End, you will find how many Feet are contained in that Piece; or you may cut off any Number of solid Feet that shall be desired; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught, by dividing the Distance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches and the Depth 9 Inches, divide the Space upon the Line of Numbers into two equal Parts, and you will find the middle Point at 15; so is 15 Inches the geometrical mean Proportional sought; then if you look for 15 upon the Line above-mentioned, you will find 7 Inches and a little above half to be the Length of a Foot.

§ III. Of unequal Squared Timber.

BY unequal Squared Timber, I mean all such as have unequal Bases; that is, such as is thicker at one End than at the other; and such are most Timber-trees when they are hewn, and brought to their Squares.

The usual Way to measure such Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is something near the Truth; but when there is a great Disproportion between the Ends of the Piece, the Error is considerable. All such Solids being the Frustums of Pyramids, the true Way of measuring them must be by Sect. VII. Chap. II. I shall give an Example or two, which I will work both by the true and false Ways, whereby you will see the Difference.

Example 1. If a Piece of Timber be 25 Inches square at the greater End, and 9 Inches square at the lesser End, and 20 Feet long; How many Feet of Timber are in that Tree?

$$\begin{array}{r}
 25 \\
 9 \\
 \hline
 \text{Sum} \quad 34 \\
 \text{Half} \quad 17 \quad \text{the Side of the Square in the Middle.} \\
 17 \\
 \hline
 119 \\
 17 \\
 \hline
 289 \\
 20 \\
 \hline
 144)5780(40.13 \\
 200 \\
 560 \\
 \hline
 128
 \end{array}$$

Answer, 40.13 Feet, by the false Way.

Y 3

By

By Rule II. Sect. VII. Chap. II.

$$\begin{array}{r}
 25 \\
 \underline{9} \\
 225
 \end{array}
 \qquad
 \begin{array}{r}
 25 \\
 \underline{9} \\
 16 \\
 \underline{16} \\
 96 \\
 \underline{16} \\
 3)256 \text{ the Square.} \\
 \underline{85.333} \\
 225 \\
 \underline{310.333} \\
 20 \\
 \underline{144)6206.660(43.101} \\
 \underline{446} \\
 146 \\
 \underline{260} \\
 116
 \end{array}$$

Answer, 43,101 Feet, by the true Way; so that there is near 3 Feet Difference.

By Scale and Compasses.

Extend from 1 to 9, that Extent will reach from 25 (the same Way) to 225, the Rectangle of the Sides of the two Bases; then the Difference between the said Sides is 15; extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that

Extent

Extent will reach from 20 (the Length) to 43.1 Feet, the Content the true Way.

Extend the Compasses from 12 to 17 (the Side of the middle Square), that Extent will reach from 20 (the Length, being twice turned over) to 40.1 Feet, the Content by the false Way.

Example 2. If a Piece of Timber be 32 Inches broad and 20 Inches deep, at the greater End, and 10 Inches broad and 6 deep, at the lesser End, and 18 Feet long; How many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap. 2.

$$\begin{array}{r} 32 \\ 20 \\ \hline 640 \end{array}$$

$$\begin{array}{r} 6 \\ 10 \\ \hline 60 \end{array}$$

$$\begin{array}{r} 38400 \\ 1 \\ \hline \end{array}$$

(195.959 mean Proportional;
640 the greater Base.
60 the lesser Base.

$$29)284$$

$$385)2300$$

$$3909)37500$$

$$39185)231900$$

$$391909)3597500$$

895.959 the Sum.
6 $\frac{1}{2}$ Height.

$$5375.754$$

$$144)5375.754(37.33$$

$$1055$$

$$477$$

$$455$$

$$23$$

Add

$$\text{Add } \left\{ \begin{array}{l} 32 \\ 10 \end{array} \right. \quad \left. \begin{array}{l} 20 \\ 6 \end{array} \right\} \text{ add.}$$

$$\text{Sum } 42 \quad 26 \text{ Sum.}$$

$$\text{Half } 21 \quad 13 \text{ half.}$$

$$13$$

$$63$$

$$21$$

273 Area in the Middle.

18 Length.

$$2184$$

$$273$$

$$144)4914(34.12$$

$$594$$

$$180$$

$$360$$

$$72$$

Feet.

$$\text{Answer. } \left\{ \begin{array}{l} \text{Content the true Way} \text{---} 37.33 \\ \text{Content the false Way} \text{---} 34.12 \end{array} \right.$$

By Scale and Compasses.

Extend the Compasses from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Base.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the lesser Base: Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the Square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, so you will find the middle Point at 195.959, the Root sought; which

is

is a mean Proportional between the greater and lesser Areas; then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiplied by 6 (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the false Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Base: Then extend from 144 to 273, that Extent will reach from 18, the Length, to 34.12, the Content the false Way.



§ IV. *Of Round Timber, whose Bases are equal.*

THE usual Way to measure Round Timber trees, is to girt them about the Middle with a String, and take the fourth Part of that Girth for the Side of a Square, by which they measure the Piece of Timber as if it was square.

But that this is an Error, I shall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which squared makes .0625; this they take for a mean Area, instead of .07958: Therefore the true Content always bears such Proportion to the Content found by the aforesaid customary false Way, as .07958 to .0625; which is nearly as 23 to 18; so that in measuring by that customary false Way, there is above the one-fifth Part lost of what the true Content ought to be.

This

This Error, though it has been so often confuted, yet it is grown so customary in all Places, that there is little Hope of my prevailing with Men that are so wedded to it, to embrace the Truth: I shall therefore, in the following Examples, shew how to work both the true Way, and also the false or customary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference or Girth, and 18 Feet long; How many Feet of Timber are contained therein?

A fourth Part of 96 is 24

24

—

96

48

—

576

Area Base:

18

—

4608

576

—

144)10368(72

1008

—

288

288

—

...

Or thus,

F. I.

2 0

2 0

—

4 0

18

—

72 0

Content the false Way, 72 Feet.

Then

Then the true Way.

$$\begin{array}{r}
 96 \\
 96 \\
 \hline
 576 \\
 864 \\
 \hline
 9216 \\
 .07958 \\
 \hline
 73728 \\
 46080 \\
 82944 \\
 64512 \\
 \hline
 \end{array}$$

733.40928 the Area by Prob. 5. § IX. Ch. 2.
18

$$\begin{array}{r}
 586727424 \\
 73340928 \\
 \hline
 \end{array}$$

144)13201.36704(91.67 Feet, the true Content.

$$\begin{array}{r}
 241 \\
 973 \\
 1096 \\
 \hline
 88
 \end{array}$$

By Scale and Compasses.

Extend from 12 to 24 (the fourth Part of the Girth) that Extent, turned twice over from 18 Feet (the Length) will at last fall upon 72 Feet, the Content the customary Way.

Extend from 42.54 to 96 (the Girth) that Extent will reach from 18 Feet, turned twice over, to 91.67 Feet, the true Content.

Example,

Example 2. If a Piece of Timber be 86 Inches Girth and 20 Feet long; How many Feet are contained therein?

The fourth Part of 86 is 21.5

F. I. P.

1 9 6

1 9 6

1 9 6

1 4 1 6

0 10 9

3 2 6 3

20

64 2 5 0

21.5

21.5

1075

215

430

462.25

20

144)9245.00(64.2

605

290

20

The Content the false Way, 64.2 Feet.

Example 2. If a Piece of Timber be 86 Inches Girth and 20 Feet long; How many Feet are contained therein?

Example

By

By the true Way.

$$\begin{array}{r}
 86 \\
 86 \\
 \hline
 516 \\
 688 \\
 \hline
 7396 \\
 .07958 \\
 \hline
 59168 \\
 36980 \\
 66564 \\
 51772 \\
 \hline
 588.57368 \\
 20 \\
 \hline
 144)11771.47360(81.74 \\
 \hline
 251 \\
 1074 \\
 667 \\
 \hline
 91
 \end{array}$$

The true Content, 81.74 Feet.

By Scale and Compasses.

Extend from 12 to 21.5, that Extent, turned twice over from 20, will reach at last to 64.2 Feet, the Content the false Way.

Extend from 42.54 to 86, that Extent, turned twice over from 20, will at last fall upon 81.74 Feet, the true Content.

These Cylindrical Proportions may be very easily wrought upon the Line of Numbers.

Z

Problem

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppose the Diameter 22.6 Inches.

As 22.6 is to 46.9, so is 1 to a fourth Number, and that to the Length of a Foot in Inches 4.3.

Extend the Compasses from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-measure, to find the Length of a Foot in Foot-measure.

Suppose the Diameter 1.88 Feet.

Then, as 1.88 is to 1.28, so is 1 to a fourth Number: And so is that to the Length of a Foot in Foot-measure, .358.

Extend the Compasses from 1.88 to 1.28, that Extent, turned twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppose the Circumference 71 Inches.

Then, as 71 is to 247.36, so is 1 to a fourth Number; and so is that to the Length of a Foot in Inches, 4.3.

Extend the Compasses from 71 to 247.36, that Extent turned twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem 4. Having the Circumference in Foot-measure, to find the Length of a Foot in Foot-measure.

Suppose the Circumference 5.92 Feet.

Then, as 5.92 is to 3.345, so is 1 to a fourth Number; and so is that to the Length of a Foot in Foot-measure, .358.

2

Extend

Extend the Compasses from 5.92 to 3 545; that Extent, turned twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Inches, 62674.

Extend the Compasses from 1.128 to 22.6, that Extent, turned twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square is equal to 1.

Problem 6. Having the Diameter in Foot-measure, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 is to 1.88, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 1.128 to 1.88, that Extent, turned twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 46.9 to 22.6, that Extent, turned twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle, whose Area is 1728.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 is to 22.6, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 13.54 to 22.6, that Extent, turned twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppose the Circumference 71, and the Length 156 Inches.

Then, as 3.545 is to 71, so is 156 to a fourth Number; and so is that to 62674, the Content in Inches.

Extend the Compasses from 3.545 to 71, that Extent, turned twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is the Circumference, when the Side of the Square is equal to 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 is to 5.92, so is 13 to a fourth Number; and so is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turned twice from 13, will fall upon 36.27 Feet, the Content.

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 is to 71, so is 156 to a fourth Number; and so is that to the Content in Feet 36.27.

Extend the Compasses from 147.36 to 71, that Extent, turned twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 147.36 is the Circumference of a Circle, whose Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 is to 71, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend the Compasses from 42.54 to 71, that Extent, turned twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle, whose Area is 144.

§ V. Of Round Timber, whose Bases are unequal.

THE usual Way to measure Round Timber (as I said before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have proved to be erroneous in Timber that is all the Way of an equal Thickness; and it must be much more so in Timber that is tapering; and the more tapering it is, the greater is the Error: For to the Error in the last Section, there is added the Error in the third Section; therefore, to measure all such Timber according to Art and Truth, such a Piece ought to be considered as a Frustum of a Cone, and should be measured by the Rules given in Section VIII. Chapter 2. by which Rules the following Examples are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the lesser End, and 36 Inches at the other End, and 24 Feet long; How many Feet of Timber are therein?

$$\begin{array}{r} 36 \\ 9 \\ \hline \text{Rect. } 324 \end{array}$$

$$\begin{array}{r} 36 \\ 9 \end{array} \left. \vphantom{\begin{array}{r} 36 \\ 9 \end{array}} \right\} \text{Subtract.}$$

$$\begin{array}{r} 27 \text{ Difference.} \\ 27 \\ \hline 189 \\ 54 \\ \hline 3 \overline{) 729} \text{ the Square.} \\ 243 \text{ One-third.} \\ 324 \text{ Rectangle add.} \\ \hline 567 \end{array}$$

.7854

567

54978

47124

39270

A mean Area 445.3218

24

17812872

8906436

144)10687.7232(74.22

607

317

292

4

Answer, 74.22 Feet.

Or thus, by Feet and Inches.

F. I.

3 0

0 9

2 3 Rect.

F. I.

2 3 Difference.

2 3

4 6

0 6 9

5 0 9 the Square.

1 8 3 One-third.

2 3 0 Rectangle added.

3 11 3 a mean Square.

Then,

F. I. P.

Then, As 14 is to 11, so is 3 11 3 to the Area.

$$\begin{array}{r}
 \hline
 7 \overline{) 43 \ 3 \ 9} \\
 \hline
 2 \overline{) 6 \ 2 \ 3} \\
 \hline
 \begin{array}{r}
 3 \ 1 \ 1 \ 6 \\
 6 \\
 \hline
 18 \ 6 \ 9 \ 0 \\
 4 \\
 \hline
 74 \ 3 \ 0 \ 0
 \end{array}
 \end{array}$$

Here, instead of dividing by 14, I divide by 7 and by 2, because twice 7 is 14.

And instead of multiplying by 24 Feet, the Length, I multiply by 6 and 4, because 6 times 4 is 24.

By Scale and Compasses this is too troublesome.

Example 2: If a Piece of Timber be 136 Inches Circumference at one End, 32 Inches Circumference at the other End, and 21 Feet long; How many Feet of Timber are contained in that Piece?

136
 32
 —

272
 408
 —

4352

136
 32
 —

104 Difference.
 104
 —

416
 1040
 —

3)10816 the Square.

3605.333 One-third.
 4352 Rectangle add.
 —

7957.333 a mean Cir. squared.
 .07958
 —

63658664
 39786665
 71615997
 55701331
 —

633 24456014 the mean Area.
 21
 —

63324456014
 126648912028
 —

13298.13576294

144)13298.13(92.34

338
 501
 693
 —

117

Answer, 92.34 Feet.

By Feet and Inches, thus,

F. I.

11 4

2 8

22 8

7 6 8

30 2 8

F. I.

8 8 Difference.

8 8

69 4

5 9 4

3)75 1 4 the Square.

25 0 5 4

30 2 8 0

55 3 1 4 the Sq. of the Circum.

F. I. P. S.

88 : 7 :: 55 3 1 4 : the mean Area.

7

11)386 9 9 4

8) 35 1 11 9

4 4 8 11 the mean Area.

7

30 9 2 5

3

Facit 92 3 7 3

§ VI. *Of the Five Regular Bodies.*

THESE Bodies may all be measured by the fourth Section of Chap. 2. except it be the Cube, or Hexaedron, which is already measured in Section I. of that Chapter.

I. *Of the TETRAEDRON.*

A Tetraedron is a Solid, contained under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whose Side is 12 Inches, the perpendicular Height 9.798 Inches.



By Sect. V. Chap. 1. the Area of the Triangle will be found 62.352; a third Part of it is 20.784, which multiplied by the perpendicular Height, the Product is 203.641632 solid Inches, the Content.

10.392 the perpendicular of the Triangle.
6 Half the Side.

62.352 Area of the Triangle.

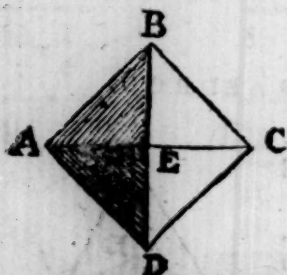
20.784 one third Part.
9.798 the perpendicular Height.

166272
187056
145488
187056

203.641632

The superficial Content is four times the Area of the Triangle, *viz.* 249.408 Inches, because there are four Triangles.

2. Of the OCTAEDRON.



The Octaedron is a Body contained under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whose Side is 12 Inches; the Content solid and superficial is required.

An Octaedron is composed of two quadrangular Pyramids joined together by their Bases; therefore, if the Area of the Base be multiplied into a third Part of the Length of both Pyramids, the Product will be the solid Content.

5.6568 a third Part of the Length.

144 Area of the Square Base.

226272

226272

56568

814 5792 the Solidity.

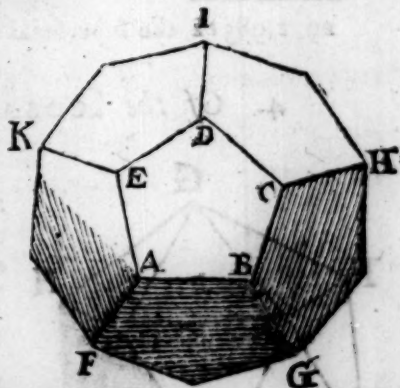
The superficial Content will be just double to that of the Tetraedron, *viz.* 498.816, because the Side of this is supposed to be equal to the Side of that, and because the Octaedron is contained under eight Triangles, and the Tetraedron but under four.

3. *Of the DODECAEDRON.*

The Dodecaedron is a solid Body, contained under twelve pentangular Planes.

Let ABCDEFGHIK be a Dodecaedron, each Side thereof being 12 Inches; the Content, solid and superficial, is required.

The Solidity of the Dodecaedron is composed of 12 pentangled Pyramids, whose Vertexes all meet in the Center.



Therefore, if we find the Solidity of one of those Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be 8.258292

30 half Sum of the Sides.

247.748760 Area of the Pentagon.

60454.4 a third Part of 13.36219 inverted:

99099504

9909950

1238744

99099

1486

1103.48783 Content of one Pyramid.

12

13241.85392 the Solidity of the Dodecaedron.

A a

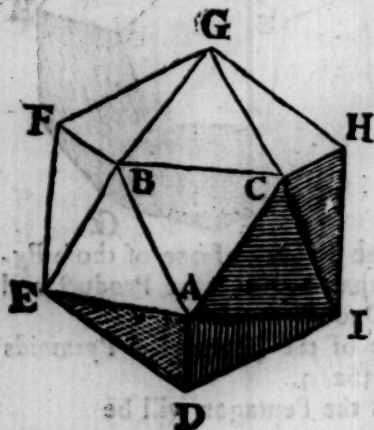
If

If the Area of the Pentagon be multiplied by 12, the Product will be the superficial Content.

$$\begin{array}{r} 247.74876 \\ 12 \end{array}$$

2972.98512 the superficial Content.

4. Of the ICOSAEDRON.



The Icosadreon is a solid Body, contained under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icosaedron, each Side thereof being 12 Inches; the Content, solid and superficial, is required.

The Icosaedron is composed of twenty triangular Pyramids, with their Vertexes all joining in the Center.

Therefore, if the solid Content of one Pyramid be multiplied by 20, the Product is the whole solid Content of the Icosaedron.

Chap. 4. *the Five Regular Bodies.* 267

10.39224 the Perpendicular of the Triangle.

6 Half the Side.

62.35344

20

1247.06880

3.0230456 third Part of the Altitude of the Pyram.

44353.26

181382736

6046091

906914

151152

9009

1209

121

188.497292

20

3769.945840 the Solidity.

The superficial Content, 1247.0688.

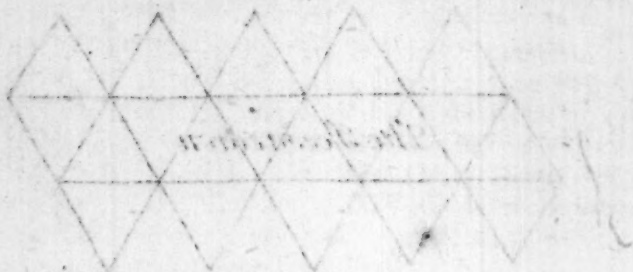
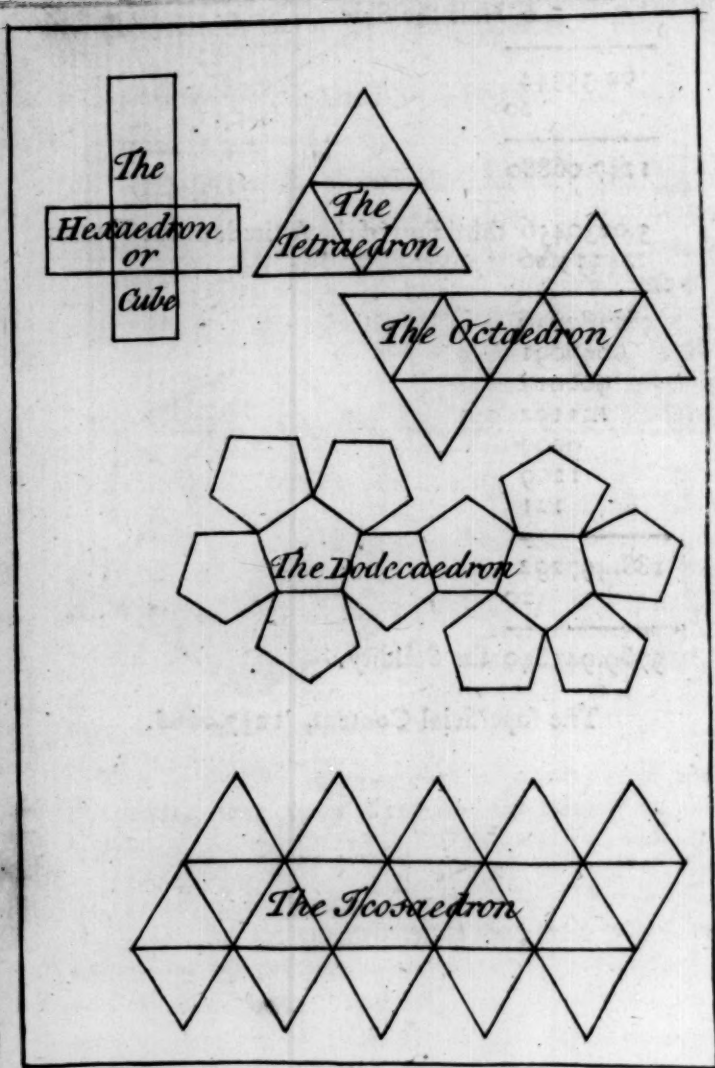


TABLE A a z The



By these Figures you may cut these Bodies in fine Pasteboard, cutting all the Lines half through, and so turn them up and glue them,

A TABLE

A TABLE shewing the Solidity and superficial Content of any of the Regular Bodies, the Side being 1, or Unity.

| The Names of the Bo- dies. | The Solidity. | Superficies. |
|----------------------------------|------------------|--------------|
| Tetraedron | 0.1178511 | 1.732051 |
| Octaedron | 0.4714045 | 3.464102 |
| Hexaedron | 1.0000000 | 6.000000 |
| Icosaedron | 2.181695 | 8.660254 |
| Dodecaedron | 7.663119 | 20.645729 |

By this Table, the Content, either superficial or solid, of any of these Bodies, may very readily be found; for all like superficial Figures are in Proportion one to another, as are the Squares of their like Sides: Therefore it will be, As the Square of 1, (which is 1) is to the superficial Content in the Table; so is the Square of the Side of the like Body, to the superficial Content of the same Body. Therefore, if the Number in the Table be multiplied by the Square of the Side given, the Product is the superficial Content required.

Again; all like Solids are in such Proportion to each other, as are the Cubes of their like Sides. Therefore it will be, As 1 (which is the Cube of 1) is to the solid Content in the Table, so is the Cube of the Side given, to the solid Content required. Therefore, if the Number in the Table be multiplied by the Cube of the given Side, the Product will be the solid Content of the same Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before); What is the Content solid and superficial?

7.663119 the tabular Number.

1728 the Cube of the Side.

61304952

15326238

53641833

7663119

13241.869632 the solid Content, nearly the same as
(before.

20.645729 the tabular Number.

144 the Square of the Side.

82582916

82582916

20645729

2972.984976 the superficial Content.

By Scale and Compasses.

Extend from 1 to 12 (the Side), that Extent being turned three times over from 7.63119, will at last fall upon 13241.86, &c. the solid Content.

And if you apply the same Extent twice from 20.645729, it will at last fall upon 2972.98, &c. the superficial Content.

Example

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Example 2. If the Side of an Octaedron be 20 Inches; What is the Content solid and superficial?

.4714045 the tabular Number.
8000 the Cube of the Side.

3771.2360000 the solid Content.

3.464102 the tabular Number.
400 the Square of the Side.

1385.640800 the superficial Content.

By Scale and Compasses.

Extend from 1 to 20, that Extent, turned three times over from .4714045, will at last fall upon 3771.236, the solid Content. The same Extent, turned twice over from 3.464, &c. will at last fall upon 1385.64, the superficial Content.

§ VII. *How to measure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you desire to find the Solidity, put the Solid into any regular Vessel, as a Tub, a Cistern, or the like, and pour in as much Water as will just cover it; then take out the Solid, and measure how much the Fall of the Water is, and so find the Solidity of that Part of the Vessel.

Example.

Example. Suppose a Piece of Wood or Stone to be measured, and suppose a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and covered with Water; then, when the Solid is taken out, suppose the Fall of the Water 14 Inches; Square 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Base; which multiplied by 14, the Depth or Fall of the Water, and the Product is 11259.49, &c. which divided by 1728, the Quotient is 6.51 Feet; and so much is the solid Content required.



C H A P. V.

Practical Questions in MEASURING.

Question 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contained therein?

$$\begin{array}{r}
 \text{F. I.} \\
 47 \quad 9 \\
 18 \quad 6 \\
 \hline
 376 \quad 0 \\
 47 \\
 23 \quad 10 \quad 6 \\
 9 \quad 0 \quad 0 \\
 4 \quad 6 \quad 0 \\
 \hline
 883 \quad 4 \quad 6
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 47 \quad 75 \\
 18.5 \\
 \hline
 23875 \\
 38200 \\
 4775 \\
 \hline
 9)883.375 \\
 \hline
 98.1
 \end{array}$$

Answer, 98 Yards 1 Foot.

Quest. 2. There is a Room, whose Length is 21.5 Feet, and the Breadth 17.5 Feet, which is to be paved with Stones, each 18 Inches square; I demand how many such Stones will pave it?

| | |
|-----------------|--------------------|
| 21.5 | 1.5 |
| <u>175</u> | <u>1.5</u> |
| 1075 | 75 |
| 1505 | 15 |
| <u>215</u> | <u>2.25</u> |
| | Area of one Stone. |
| 2.25)376.25(167 | |
| <u>1512</u> | |
| 1625 | |
| <u>50</u> | |

Answer, 167 Stones.

Quest. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad, to be hung with Tapestry that is Ell-broad; I desire to know how many Yards will hang the said Room?

From the Content of the Room, subtract the Content of the Windows, and divide the Remainder by the square Feet in a Yard of Tapestry.

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| | | | |
|-------|--------|----------|-------|
| 3.75 | 109 75 | Length. | 5.75 |
| 3 | 92.5 | Breadth. | 6.5 |
| <hr/> | | | <hr/> |
| 11.25 | 54875 | | 2875 |
| | 21950 | | 3450 |
| | 98775 | | <hr/> |

1015.1875 Content of the Room. 37.375
 74.75 Content of the Wind. sub. 2

11.25)940.4375(83.59

4043
 6687
 10925

500

Answer, 83.59 Yards.

Quest. 4. If the Axis of a Globe be 27.5 Inches;
 I demand the Content solid and superficial.

3.1416
 27.5

157080
 219912
 62832

86.39.400 the Circumference.

86.394

86.394 the Circumference.

27.5 the Diameter.

 431970
 604758
 172788

6)2375.8350 the superficial Content.

395.9725 a sixth Part.

27.5

 19798625
 27718075
 7919450

1.889.24375

 Answer, $\left\{ \begin{array}{l} 6.3 \text{ Feet solid.} \\ 16.49 \text{ Feet superficial.} \end{array} \right.$

Quest. 5. There is the Frustrum of a Globe, the Diameter of whose Base is 24 Inches, and the Altitude thereof is 19 Inches; What is the Content solid and superficial?

Find the Superficies as is directed in Page 188, and 24 the Solidity by the first Theorem in Page 190.

| | | | |
|-------|-------|----------|-------|
| 24 | .7854 | .7854 | 20 |
| 24 | 567 | 400 | 20 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 96 | 47124 | 314.1600 | 400 |
| 48 | 54978 | | |
| <hr/> | <hr/> | | |
| 576 | 39270 | | |

 452.3904 } add.
 314.16

766.5504 the Curve Superficies.

452.3904 the Base add.

 1218.9408 } the whole superficial Content in Inches.

$$12 \times 12 = 144$$

3

432

100 the Square of the Alt. add.

532 the Sum.

10 multiply by the Alt.

5320

5236 multiply.

31920

15960

10640

26600

2785.5520 the Solidity in Inches.

Quest. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; How many Tons of Timber are contained in that Tree?

F. I.

4)18 6 the Girth.

4 7 6 a 4th Part.

4 7 6

18 6 0

2 8 4 6

2 3 9

21 4 8 3

Here I multiply by 6 and by 4, because 6 times 4 is 24.

128 4 1 6

4

4|0)51|3 4 6 0

12 33

Answer, 12 Tons 33 Feet 4 Inches 6 Parts.

B b

Note,

Note, That 40 Feet of Timber is a Ton, and 50 Feet a Load.

Note also, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-wood, that is, 128 cubical Feet.

Quest. 7. There is a Cellar to be dug by the Floor, whose Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar?

| | | |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|--------------|
| F. | I. | |
| 33 | 7 | the Length. |
| 18 | 9 | the Breadth. |
| <div style="display: flex; justify-content: space-between;"> 264 33 </div> <div style="display: flex; justify-content: space-between;"> 16 9 6 </div> <div style="display: flex; justify-content: space-between;"> 8 4 9 </div> <div style="display: flex; justify-content: space-between;"> 9 0 0 </div> <div style="display: flex; justify-content: space-between;"> 1 6 0 </div> | | |
| <div style="display: flex; justify-content: space-between;"> 629 8 3 </div> <div style="display: flex; justify-content: space-between;"> 5 9 0 </div> | | |
| <div style="display: flex; justify-content: space-between;"> 3148 5 3 </div> <div style="display: flex; justify-content: space-between;"> 314 10 1 </div> <div style="display: flex; justify-content: space-between;"> 157 5 0 </div> | | |
| <div style="display: flex; justify-content: space-between;"> 324)3620 8 4(11 </div> <div style="display: flex; justify-content: space-between;"> 380 </div> <div style="display: flex; justify-content: space-between;"> 56 </div> | | |

Answer, 11 Floors 56 Feet.

Note, That 18 Feet square and a Foot deep is a Floor of Earth, that is, 324 solid Feet.

Quest.

Quest. 8. There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; How many Squares of Tiling are contained therein?

| F. | I. | |
|-------|----|-----|
| 48 | 9 | |
| 35 | 6 | |
| <hr/> | | |
| 240 | | |
| 144 | | |
| 24 | 4 | 6 |
| 17 | 6 | 0 |
| 8 | 9 | 0 |
| <hr/> | | |
| 17 | 30 | 7 6 |

Answer, 17 Squares 30 Feet.

Quest. 9. There is a Cone, whose Diameter at the Base is 42 Inches, and the perpendicular Height 94 Inches, and it is required to cut off two solid Feet from the Top-end thereof; I demand what Length upon the Perpendicular must be cut off?

| | | |
|---------------|-------|------------------|
| 42 | 1728 | 94 |
| 42 | 2 | 94 |
| <hr/> | <hr/> | <hr/> |
| 84 | 3456 | 376 |
| 168 | | 846 |
| <hr/> | | <hr/> |
| 1764 | | 8836 Square. |
| 7854 | | 94 |
| <hr/> | | <hr/> |
| 7056 | | 35344 |
| 8820 | | 79524 |
| 14112 | | <hr/> |
| 12348 | | 830584 the Cube. |
| <hr/> | | |
| 13854456 | | |
| 94 | | |
| <hr/> | | |
| 55417824 | | |
| 124690104 | | |
| <hr/> | | |
| 3)130231.8864 | | |
| <hr/> | | |

43410.6288

All the solid Bodies are in triplicate Reason of their homologous Sides, by *Eucl.* 12, 12; 12, 18; and 11, 33: Therefore it will be,

Solidity of the Cone. Cub. Alt. Solidity of 2 Feet.

43410.6288 : 830584 : : 3456 :

4983504
4152920
3322336
2491752

434106288)2870498304(66124 the Cube of the
(Length.

265860576
5396803
1055740
187528

06124

$$\begin{array}{r} 66124 \overline{) 40.43} \\ 64 \end{array}$$

2124000 Resolvend.

$$\begin{array}{r} 12 \\ 48 \end{array}$$

492 Divisor.

$$\begin{array}{r} 120 \\ 4800 \end{array}$$

48120 Divisor.

$$\begin{array}{r} 64 \\ 1920 \\ 19200 \end{array}$$

1939264 Subtrahend.

184736000 Resolvend.

$$\begin{array}{r} 1212 \\ 489648 \end{array}$$

4897692 Divisor.

$$\begin{array}{r} 27 \\ 10908 \\ 1468944 \end{array}$$

147003507 Subtrahend.

$$\begin{array}{r} 37732493 \end{array}$$

Answer, The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

B b 3

If

If two Feet were to be cut off from the Bottom, or greatest End, then from 43410.6288 subtract 3456, and the Remainder is 39954.6288. Then say,

$$43410.6288 : 830584 :: 39954.6288$$

830584

1598185152

3196370304

1997731440

11986388640

3196370304

$$13410.6288)33185675407.2192(764459(91.4$$

.....

729

279823524

19359751

1995500

259075

42022

2952

35459

27

243

2457

271

243

24571

10888000

273

24843

248703

Answer. It must be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Quest.

Quest. 10. If a square Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Base be 21 Inches, and the Side of the Square of the lesser Base be 3 Inches; How far must I measure from the greater End, to cut off 3 solid Feet?

First find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18: Then,

Diff. Length. great. Length.

As 18 : 12 :: 21 : 14.

So I find the whole Length of the Pyramid to be 14 Feet, or 168 Inches.

The solid Content of the whole Pyramid is 24696 Inches, and the solid Content of 5 Feet is 8640; which subtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

24696 : 4741632 :: 16056 :

3082752, whose Cube Root is 145.54; subtract this Root from 168 (the Length) and there remain 22.46 Inches, which is the Length of 5 solid Feet at the great End.

Quest. 11. Three Men bought a Grinding-stone of 40 Inches diameter, which cost 20 Shillings; of which Sum the first Man paid 9 Shillings, the second 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man must grind down, proportionable to the Money he paid?

All Circles are in duplicate Reason of their Diameter, by *Euc.* 12, 2.

Square the Semidiameter, which makes 400. Then

s. Square. s.

20 : 400 :: 9 : 180.

This 180 is the Square of the Semidiameter of the Circle belonging to the first Man.

And,

$$\begin{array}{ccc} & \text{s.} & \text{s.} \\ \text{And, } 20 : 400 :: 6 : 120. \end{array}$$

This 120 is the Square of the Semidiameter of the Circle belonging to the second.

$$\begin{array}{ccc} & \text{s.} & \text{s.} \\ \text{And, } 20 : 400 :: 5 : 100. \end{array}$$

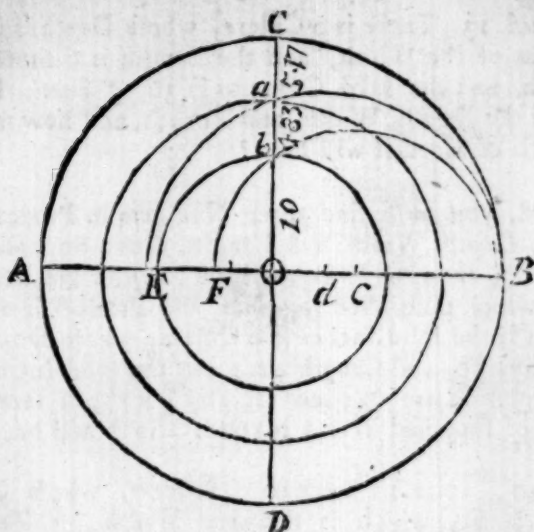
This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400 (the Square of the Semidiameter of the Stone) subtract 180; and there remains 220, whose square Root is 14.83 Inches; which subtracted from 20 Inches, (the Semidiameter), there remain 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which must be ground down by the first.

Then, from 220 subtract 120, and there remains 100, whose Square Root is 10; subtract that from 14.83, and there remain 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the second Man. The third must grind down the Remainder, which is 10 Inches, the square Root of 100.

This Question may very easily and speedily be performed geometrically, as in the annexed Scheme.

First,



First, upon the Center \odot strike the Circle ABCD, and cross it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A \odot (which suppose 20) in Proportion to 9s. 6s. and 5s. (the several Sums paid by the three Men) by the Points E and F; so shall AE be 9, EF 6, and F \odot 5: Then divide EB into two equal Parts in d, and upon d, as a Center, strike the Semicircle E a B; and divide FB into two equal Parts in c, and upon c, as a Center, with the Radius cF, strike the Semicircle FbB: So have you the Semidiameter \odot C divided into three such Parts as the Stone ought to be divided; and Circles, struck through those Points, will shew how much each Man must grind for his Share.

Quest. 12. A Gard'ner he had an upright Cone,
Out of which should be cut him a Rolling-stone,
The biggest that e'er it could make:

The Mason he said, That there was a Rule
For such sort of Work, but he had a thick Skull:

Now help him for Pity's Sake.

Answer, It must be cut at one-third Part of the Altitude.

Quest.

Quest. 13. There is a Cistern, whose Depth is seven Tenths of the Width, and the Length is 6 times the Depth, and the solid Capacity is 367.5 Feet; I demand the Depth, Width, and Length, and how many Bushels of Corn it will hold?

First, you must find three Numbers in Proportion to the Depth, Width, and Length, thus: Suppose the Depth 7, then the Width will be 10, and the Length 42; which multiplied together, the Product is 2940, which is the solid Inches in a Cistern, whose Depth is 7, Width 10, and Length 42. But the solid Inches in the Question are 635040 ($= 367.5 \times 1728$) then the Cube of supposed Width is 1000. So it will be,

2940 : 1000 :: 635040 : 216000, whose Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiplied together, the Product will be 635040. If these solid Inches be divided by 2150.42, and the Quotient is $295 \frac{66610}{215042}$ Bushels, or 36 Quarters 7 Bushels 1 Peck 4 Pints. And so much will the Cistern hold.

Quest. 14. Suppose, Sir, a Bushel be exactly round, Whose Depth being measur'd, 8 Inches is found; If the Breadth 18 Inches and half you discover, This Bushel is legal all *England* over. But a Workman would make one of another Frame; Sev'n Inch and a half must be the Depth of the same; Now, Sir, of what Length must the Diameter be, That it may with the former in Measure agree?

18.5
18.5

925
1480
185

342.25 the Square.
.7854

136900
171125
273800
239575

268.803150
8

2150.425200 the solid Inches in a Bushel.

7.5)2150.4252(286.72336
...

650
504
542
175
252
270
450

...

$$\begin{array}{r} .7854 \overline{) 286.72336} \\ \hline \end{array}$$

$$\begin{array}{r} 51103 \\ \hline \end{array}$$

$$\begin{array}{r} 39793 \\ \hline \end{array}$$

$$\begin{array}{r} 5236 \\ \hline \end{array}$$

$$\begin{array}{r} .365 \overline{) 0.666(19.107} \\ \hline \end{array}$$

$$\begin{array}{r} 29 \overline{) 265} \\ \hline \end{array}$$

$$\begin{array}{r} 261 \\ \hline \end{array}$$

$$\begin{array}{r} 381 \overline{) 406} \\ \hline \end{array}$$

$$\begin{array}{r} 381 \\ \hline \end{array}$$

$$\begin{array}{r} 38207 \overline{) 256600} \\ \hline \end{array}$$

Answer, The Diameter must be 19.107 Inches, if the Depth be 7.5 Inches.

Quest. 15. In the Midst of a Meadow well stored with Grass,

I took just an Acre to tether my As;

How long must the Cord be, that feeding all round,

He may'nt graze less nor more than his Acre of Ground?

By Problem 10. Section IX. Chap. 1. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.

66 Feet, the Breadth of an Acre.

$$\begin{array}{r} 3960 \\ \hline \end{array}$$

$$\begin{array}{r} 3960 \\ \hline \end{array}$$

43560 the square Feet in an Acre.

As

As 1 : 1.2732 :: 43560

43560

763920

63660

38196

50928

55460.5920(235.5 Diameter.

4

117.75 half.

43)154

129

465)2560

2325

4705)23559

23525

34

Answer, The Cord must be 147 Feet and 9 Inches.

Quest. 16. A Maltster has a Kiln, that is 16 Feet 6 Inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much square the new one must be?

$$\begin{array}{r}
 16.5 \\
 16.5 \\
 \hline
 825 \\
 990 \\
 165 \\
 \hline
 272.25 \text{ the Area of the old one.} \\
 3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \dot{8} \dot{1} \dot{6} . \dot{7} \dot{5} (28.57 \\
 4 \\
 \hline
 48)416 \\
 384 \\
 \hline
 365)3275 \\
 2825 \\
 \hline
 3707)45000 \\
 39949 \\
 \hline
 5051
 \end{array}$$

Answer, The Side of the new one must be 28 Feet and near 7 Inches.

Quest. 17. If a round Cistern be 26.3 Inches diameter, and 52.5 Inches deep; How many Inches diameter must a Cistern be to hold twice the Quantity, the Depth being the same? And how many Ale Gallons will each Cistern hold?

$$\begin{array}{r}
 26.3 \\
 26.3 \\
 \hline
 789 \\
 1578 \\
 526 \\
 \hline
 691.69 \text{ Square.} \\
 2 \\
 \hline
 1383.38(37.19 \\
 9 \\
 \hline
 67)483 \\
 469 \\
 \hline
 741)1438 \\
 741 \\
 \hline
 7429)69700 \\
 66861 \\
 \hline
 2839
 \end{array}$$

The Diameter of the greater is 37.19 Inches.

691.69 the Square of the lesser Cif-
 .7854 (tern's Diameter.

$$\begin{array}{r} 276676 \\ 345845 \\ 553352 \\ 484183 \end{array}$$

543.253326 Area of the Base
 52.5

$$\begin{array}{r} 2716266630 \\ 1086506652 \\ 2716266630 \end{array}$$

28520.7996150 solid Content in Inches.

282)28520.799(101.137 Gallons.

$$\begin{array}{r} 320 \\ 387 \\ 1059 \\ 2139 \\ 165 \end{array}$$

Note, That 282 solid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

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You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

$$\begin{array}{r}
 359.05 \overline{) 1383 \ 38} \qquad (3 \ 853 \\
 \underline{306230} \qquad \qquad \underline{52.5} \\
 18990 \qquad \qquad 19265 \\
 \underline{1037} \qquad \qquad \underline{7706} \\
 19265
 \end{array}$$

The Content of the greater, 202.2825 Gallons.

Quest. 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches; How many Ale Gallons are contained therein?

| | | |
|-----------|-----------|----------|
| 25 | 32 | 359 |
| <u>25</u> | <u>32</u> | <u>3</u> |
| 125 | 64 | 1077 |
| <u>50</u> | <u>96</u> | |
| 625 | 1024 | |

1024 Square of the Bung Diameter.
 1024 the same.
 625 Square of the Head Diameter.

$$\begin{array}{r}
 1077 \overline{) 2673} \qquad (2 \ 48 \\
 \underline{5190} \qquad \qquad \underline{40} \\
 8820 \qquad \qquad 99.20 \\
 \underline{204}
 \end{array}$$

Answer, 99.2 Gallons.

Otherwise you may find a mean Diameter, and work by Scale and Compasses, thus: Subtract 25 from 32, and there remains 7, which multiplied by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compasses from 18.95 to 29.9, that Extent turned twice from 40 (the Length) will fall upon 99.6 Gallons; something more than before.

Quest. 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then (by *Euc. 11. 33.*)

217 : 8000 :: 1000 : 36870 645, whose Cube-Root is 33.28 Inches, the Length of the Stone weighing 2000 Pounds. Then say,

$$\begin{array}{l} 20 : 33.28 :: 15 : 24.96 \\ 20 : 33.28 :: 8 : 13.312 \end{array}$$

Answer, $\left\{ \begin{array}{l} \text{The Length} \quad 33.28 \\ \text{The Breadth} \quad 24.96 \\ \text{The Thickness} \quad 13.312 \end{array} \right\} \text{Inches.}$

Quest. 20. If an Iron Bullet, whose Diameter is 4 Inches, weighs 9 Pounds; What will be the Weight of another Bullet (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729. Then (by *Euc. 12. 18.*)

$$\begin{array}{l} \text{lb.} \qquad \qquad \text{lb.} \\ 64 : 9 :: 729 : 102.515. \end{array}$$

Answer, It weighs 102 $\frac{\text{lb.}}{\text{oz.}} \frac{\text{dr.}}{4}$ *ferè.*

Quest.

Quest. 21. There is a square Pyramid of Marble; each Side of its Base is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like square Pyramid, each Side of whose Base is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000.
Then (by *Euc.* 12.12.)

$$\begin{array}{ccc} \text{lb.} & & \text{lb.} \\ 125 : 12.25 :: 27000 : 2646. \end{array}$$

Answer, The Weight is 2646 Pounds.

Quest. 22. There is a Ball or Globe of Marble, whose Diameter is 6 Inches, and its Weight 11 Pounds; What will be the Diameter of another Globe of the same Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

$$\begin{array}{ccc} \text{lb.} & & \text{lb.} \\ 11 : 216 :: 500 : 981.1818. \end{array}$$

Whose Cube Root is 21.4 Inches, the Diameter sought.

Quest. 23. There is a Frustum of a Pyramid, whose Bases are regular Octagons; each Side of the greater Base is 21 Inches, and each Side of the lesser Base is 9 Inches, and its Length is 15 Feet; I demand how many solid Feet are contained therein?

4.8284 the tabular Number. Page 88,
237 the Square of a mean Side.

| | | |
|-----------|-------|-------|
| 337988 | 21 | 12 |
| 144852 | 9 | 12 |
| 96568 | <hr/> | <hr/> |
| | 189 | 3)144 |
| 1144.3308 | 48 | <hr/> |
| 15 | <hr/> | 48 |
| | 237 | |
| 57216540 | | |
| 11443308 | | |

144) 17164.9620 (119.2

276
1324
289

1

Answer, 119.2 solid Feet.

Quest. 24. There is a Frustum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches; and the Length or Height is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustum?

First, Find the Length of the whole Cone, thus:

From 36
Subtr. 20

16 : 215 :: 36 : 483.75

So the Length of the whole Cone is 483 $\frac{3}{4}$ Inches.

Then

Then find the Content of the whole Cone.

| | |
|----------------------|----------------------------|
| 36 | 1017.8784 |
| 36 | 52.161 |
| <hr/> | <hr/> |
| 216 | 10178784 |
| 108 | 6107270 |
| <hr/> | <hr/> |
| 1296 | 101788 |
| .7854 | 20357 |
| <hr/> | <hr/> |
| 5184 | 5089 |
| 6480 | <hr/> |
| 10368 | 1728)164132.88(94.98 Feet, |
| 9072 | <hr/> |
| <hr/> | <hr/> |
| 1017.8784 Area Base. | 8612 |
| | 17008 |
| | <hr/> |
| | 14568 |
| | <hr/> |
| | 744 |

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the Top-part that is wanting.

.7854 the Area of Unity.
400 the Square of 20.

3)314.1600 Area of the lesser Base,

104.72 a third Part.
268.75 Altitude of the Top-part.

52360
73304
83776
62832
20944

1728)23143.5000(16.28 Feet.

10863
4955
14990
1166.

Content of the Whole 94.98 Feet.
Content of the Top-piece 16.28

Content of the Frustum 78.7

Quest. 25. If the Top-part of a Cone contains 26171 solid Inches, and 200 Inches its Length, and the lower Frustum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base?

| | | |
|--------|--------|----------|
| 200 | 159610 | } add. |
| 200 | 26171 | |
| <hr/> | | |
| 40000 | 185781 | the Sum. |
| 200 | | |
| <hr/> | | |
| 399990 | | |

26171

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26171 : 8000000 :: 185781 : 56789881, whose
Cube Root is 384 3 Inches, the Length of the whole
Cone.

Then find the Diameter of the lesser Base, thus:

$$200 \overline{) 26171}$$

$$\begin{array}{r} 130.855 \\ 3 \end{array}$$

392.565 Area of the lesser Base.

Then, by Prob. 10. Sect. IX. Chap. 1.

$$1 : .12732 :: 392.565$$

$$\begin{array}{r} 1.2732 \\ \hline 785130 \\ 1177695 \\ 2747955 \\ 785130 \\ 392565 \end{array}$$

$$\begin{array}{r} 499.8137580(22.35 \\ 4 \end{array}$$

$$\begin{array}{r} 42 \overline{) 99} \\ 84 \end{array}$$

$$\begin{array}{r} 443 \overline{) 1581} \\ 1329 \end{array}$$

$$\begin{array}{r} 4465 \overline{) 25237} \\ 22325 \\ \hline 2912 \end{array}$$

Again,

Lesser Len. Less. Diam. Gr. Leng. Gr. Diam.
Again; 200 : 22.35 : : 384.3 : 42.94.

Inches.

Answer, { The Length of the whole Cone 384.3
The Diameter of the greater Base 42.94
The Diameter of the lesser Base 22.35

Quest. 26. There is a Frustum of a Cone, whose solid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears such Proportion to the lesser as 5 to 2; I demand the Diameters?

$$5 \times 5 = 25$$

$$2 \times 2 = 4$$

$$5 \times 2 = 10$$

$$3)12$$

$$4)20(5 \text{ Feet.}$$

The Sum 39

These 5 Feet are the Triple
of a mean Area.

Then, 1 : 1.27324 :: 5 : 6.3662.

So the triple Square of a mean Diameter is 6.3662.

Then, 39 : 6.3662 :: 25 : 4.080897.

This 4.080897 is the Square of the greater Diameter, whose Square Root is 2.020123 Feet; which is 24.24147 Inches. Then,

5 : 24.24147 :: 2 : 9.69659

So the greater Diameter is 24.24147, and the lesser Diameter is 9.69659 Inches.

Quest. 27. There is a Room of Wainscot 120 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high (being girt over the Mouldings;) there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-boards and Top and Bottom boards of each

Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainscot are contained in that Room?

$$\begin{array}{r} \text{F. I.} \\ 129 \quad 6 \\ 16 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 782 \quad 0 \\ 129 \\ \hline 64 \quad 9 \\ 32 \quad 4 \quad 6 \\ \hline 2169 \quad 1 \quad 6 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 7 \quad 3 \\ 4 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \quad 0 \\ 3 \quad 7 \quad 6 \\ \hline 32 \quad 7 \quad 6 \\ 16 \quad 3 \quad 9 \text{ half.} \\ \hline 48 \quad 11 \quad 3 \\ 2 \\ \hline 97 \quad 10 \quad 6 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 3 \quad 3 \\ 7 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 22 \quad 9 \\ 11 \quad 4 \quad 6 \text{ half.} \\ \hline 34 \quad 1 \quad 6 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 24 \quad 6 \\ 1 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \quad 6 \\ 18 \quad 4 \quad 6 \\ \hline 42 \quad 10 \quad 6 \\ 2 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 7 \quad 3 \\ 5 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \quad 3 \\ 3 \quad 7 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 3 \quad 6 \\ 7 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 39 \quad 10 \quad 6 \\ 2 \\ \hline \end{array}$$

$$79 \quad 9 \quad 0$$

$$\begin{array}{r} 24 \quad 6 \\ 79 \quad 9 \\ \hline 104 \quad 3 \end{array} \left. \begin{array}{l} \\ \end{array} \right\} \text{add.}$$

D d

The

| | | | |
|--------------------------------|------|----|---|
| The Content of the Room | 2169 | 1 | 6 |
| The Shutters, at Work and half | 97 | 10 | 6 |
| The Door, at Work and half | 34 | 1 | 6 |
| The Cheek-boards, &c. | 85 | 9 | 0 |

| | | | |
|---------------------------------------------|------|----|---|
| The Sum | 2386 | 10 | 6 |
| The Window lights and } Door-case deduct | 104 | 3 | 0 |

9) 2282 7 6

253 5

Answer, 253 Yards 5 Feet.

Quest. 28. There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiplied together, the Product will be 1600, and the Cube of 2 is 8: Then say,

$$1600 : 8 :: 18225 : 91125.$$

Then the Cube Root of 91125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5, the Height; and 8 times 22.5 is 180, the Length.

Quest. 29. There is a May-pole, whose Top-end was broken off by a Blast of Wind, and the Top-end, in falling, struck the Ground at 15 Feet Distance from the Foot of the May-pole, the broken Piece was 39 Feet; Now I demand the Length of the May-pole?

By *Euc.* 1. 47, the Square of the Hypotheneuse of a right-angled Triangle is equal to the Sum of the Squares of the Base and Perpendicular.

Therefore, from the Square of 39 subtract the Square of 15, the Square Root of the Remainder is the Piece standing; to which add the Piece broken off, and you have the whole Length.

$$\begin{array}{r}
 39 \\
 39 \\
 \hline
 351 \\
 117 \\
 \hline
 1521 \\
 225 \\
 \hline
 \dots \\
 1296/36 \\
 9 \\
 \hline
 66)396 \\
 396 \\
 \hline
 \dots
 \end{array}$$



| | |
|-------------------------|----------|
| The Piece standing is | 36 Feet. |
| The Piece broken off is | 39 Feet. |
| | <hr/> |
| The whole Length | 75 |

Question 30.

A May-pole there was, whose Height I would know,
 The Sun shining clear, strait to work I did go:
 The Length of the Shadow, upon level Ground,
 Just sixty-five Feet, when measur'd, I found;
 A Staff I had there, just five Feet in Length;
 The Length of its Shadow was four Feet One-tenth:
 How high was the May pole, I gladly would know?
 And it is the Thing you're desir'd to show.

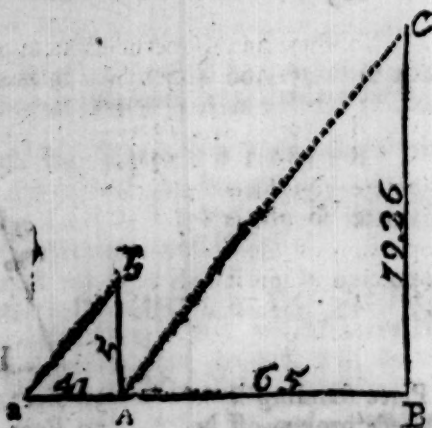
By Eucl. 6. 4.

$$aA : Ab :: AB : BC.$$

That is,

$$4.1 : 5 :: 65 : 79.26.$$

So I find the Height of the May-pole to be 75 Feet and a little above 3 inches.



Here AB represents the Length of the Shadow of the May-pole, and BC the May-pole; a A the Shadow of the Staff, and Ab the Staff.

Quest. 31. What will be the Diameter of a Globe, when the Solidity and superficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the superficial Content, we must multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the

the Product is the Solid Content; therefore, because .5236 is a sixth Part of 3.1416, we must take 6 for the Diameter sought. For if 3.1416 be multiplied by the Square of 6; viz. by 36, the Product will be 113.0976; and if .5236 be multiplied by the Cube of 6; viz. by 216, the Product is likewise 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Answer.

Quest. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Because the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

$$8 : 18 :: 6 : 13.5.$$

So the Diameter sought is $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

$$18 : 8 :: 6 : 4\frac{2}{3}.$$

So then the Diameter will be $2\frac{2}{3}$.

The Reason of these Operations, both in this and the last Question, is from Algebra.

Quest. 33. There are three Grenado-shells, of such Capacity, that the second Shell will just lie in the Concavity of the first, and the third in the Concavity of the second. The Solidity of the Metal of the first Shell is equal to its Concavity, and the Solidity of the Metal to the second, to the Concavity, is as 7 to 5; and the Solidity of the third, or least Shell's Metal, to its Concavity, is as 9 to 4. Now, supposing the Diameter of the first, or greatest Shell, to be 16.

D. d 3

Inches,

Inches, and allowing every solid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two lesser Shells; and the Thickness and Solidity of Metal of every Shell; and also the Weight of every Shell?

The Cube of 16 is 4096; then,

$$1 : .5236 :: 4096 : 2144.6656.$$

The half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as also the Concavity.

$$.5236 : 1 :: 1072.3328 : 2048.$$

The Cube Root of 2048 is 12.699, which is the Diameter of the second Shell.

The Sum of 7 and 5 is 12; then,

$$12 : 5 :: 1072.3328 : 446.805.$$

This 446.805 is the solid Content of the Concavity of the second.

$$.5236 : 1 :: 446.805 : 853.333.$$

The Cube Root of 853.333 is 9.485, the Diameter of the least Shell.

The Sum of 9 and 4 is 13; then,

$$13 : 4 :: 446.805 : 137.47846.$$

This 137.47846 is the solid Content of the Concavity of the third.

$$.5236 : 1 :: 137.47846 : 262.5639.$$

The Cube Root of 262.5639 is 6.4034, the Diameter of the least Shell's Concavity.

From 16 the Diameter of the greatest,
Subtr. 12.699 the Diameter of the second.

Rem. 3.301

Half is 1.65 the Thickness of Metal of the greatest.
From

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From 12.699 the Diameter of the second,
Subtr. 9.485 the Diameter of the least.

Rem. 3.214

Half is = 1.607 the Thickness of Metal of the second.

From 9.485 the Diameter of the least,
Subtr. 6.403 the Diameter of the Concavity.

Rem. 3.082

Half is = 1.521 the Thickness of Metal of the least.

The Metal of the greatest is 1072.33 solid Inches;
which divide by 4 (because every solid Inch is a
Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the second is 625.52 solid Inches;
which divided by 4, the Quotient is 156.38 Pounds,
the Weight of the second.

The Metal of the least Shell is 309.32 solid Inches;
which divided by 4, the Quotient is 77.33 Pounds,
the Weight of the least.

| | | | | |
|--------------|---|--------------|--------|-----------|
| The Diameter | { | second Shell | 12.699 | } Inches. |
| of the | | least Shell | 9.485 | |

| | | | | |
|---------------|---|----------|-------|-----------|
| The Thickness | { | greatest | 1.65 | } Inches. |
| of the Metal | | second | 1.607 | |
| of the | | least | 1.541 | |

| | | | | |
|------------|---|----------|--------|-----------|
| The Weight | { | greatest | 268.08 | } Pounds. |
| | | second | 156.38 | |
| | | least | 77.33 | |

Chapter 2

Of the Nature and Properties of the Mind

Section 1. The Mind is a Substance

Section 2. The Mind is a Simple Substance

Section 3. The Mind is a Thinking Substance

Section 4. The Mind is a Free Substance

Section 5. The Mind is a Simple and Indivisible Substance

Section 6. The Mind is a Substance that is not Extended

Section 7. The Mind is a Substance that is not Divisible

Section 8. The Mind is a Substance that is not Movable

Section 9. The Mind is a Substance that is not Changeable

Section 10. The Mind is a Substance that is not Destroyable

Section 11. The Mind is a Substance that is not Dependent

Section 12. The Mind is a Substance that is not Limited

Section 13. The Mind is a Substance that is not Measurable

Section 14. The Mind is a Substance that is not Quantifiable

Section 15. The Mind is a Substance that is not Numerable

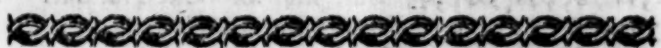
Section 16. The Mind is a Substance that is not Calculable

Section 17. The Mind is a Substance that is not Estimable

A SHORT



A SHORT APPENDIX.



§ I. Of GAUGING.



SHALL not here give the whole Art of Gauging (there being several Books of that Art already in Print, written by better Hands); but shall only lay down some short practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Vessel upon Occasion.



PROBLEM I.

To find the several Multipliers, Divisors, and Gauge-points, belonging to the several Measures now used in England.

282) 1.0000(.003546 Multiplier for Ale Gallons.
231) 1.0000(.004329 Multiplier for Wine Gallons.
268 8) 1.000(.0037202 Multiplier for Corn Gallons.
2150.42) 1.000(.00046502 Multipl. for Corn Bushels.

So, if the solid Inches in any Vessel be multiplied by the said Multipliers, the Product will be Gallons
in

in the respective Measures; or dividing by the Divisors 282.231, or 268.8, the Quotient will likewise be Gallons.

Note. That 282 solid Inches is a Gallon of Ale or Beer-measure; 231 solid Inches is a Gallon of Wine-measure; 268.8 solid Inches is a Gallon, and 2150.42 solid Inches is a Bushel of Corn-measure.

For circular Areas, the following Multipliers and Divisors are to be used.

282).785398(.002785 Multipliers for Ale Gallons.
 231) 785398(.003399 Multiplier for Wine Gallons.
 .785398)282(.35905 Divisor for Ale Gallons.
 .785398)231(.294.12 Divisor for Wine Gallons.
 .785398)2150.42(.2738 Divisor for Corn Bushels.

The Square Root of the Divisor is the Gauge-point.

| | | | |
|-------------------------------------------------|---|------------------|-------|
| The Gauge-point for Squares in | { | Ale-measure, is | 16.79 |
| | | Wine-measure, is | 15.19 |
| | | Malt-bushel, is | 46.36 |
| The Gauge-point for circular Ri- gures in | { | Ale-measure, is | 18.95 |
| | | Wine-measure, is | 17.15 |
| | | Malt-bushel, is | 52.32 |

PROBLEM II.

To find the Area in Ale or Wine Gallons, of any rectilineal plain Figure, whether Triangular, Quadrangular, or Multangular.

TO resolve this Problem, you must, by Chap. I. Part II. find the Area in Inches, and then bring it to Gallons, by dividing that Area in Inches by the proper Divisor; viz. by 282 for Ale, or by 231 for Wine; or else by Multiplication, by .003546 for Ale, or by .004329 for Wine; and the Quotient or Product will be the Area.

Example.

Example. Suppose a Back or Cöbler in the Form of a Parallelogram, or long Square, 250 Inches in Length, and 84.5 Inches in Breadth; What is the Area in Ale or Wine Gallons?

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches, which divide by 282, and the Quotient is 74.9 Gallons of Ale; or multiplied by .003546, the Product is 74.20928 Gallons, nearly the same; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.44 Gallons of Wine.

By Scale and Compasses.

Extend the Compasses from 282 to 250, that Extent will reach from 84.5 to 74.9. And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Note, The Areas of all Superficies are always to be understood to be 1 Inch deep; otherwise it could not be said, that the Area of such a Parallelogram, Circle, &c. is so many Gallons.

Having found the Area of a Back or Cöbler, the next Thing will be to find out the true Dipping or Gauging-place in that Back, that so the true Quantity of Worts may be computed at any Depth; which may be thus done.

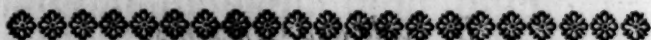
1. When the Bottom of the Back is covered all over (of any Depth) with Worts, or other Liquor, then dip it in eight or ten several Places (more or less, according to the Largeness of the Back), as remote and equally distant from each other as you can well do, noting down the Wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all those Dips by the Number of Places you dipped in, and the Quotient will be the mean Wet of all those Dips.

3. Lastly

3. Lastly, find out such a Place by the Side of the Back (if you can) that just wets the same with that mean Dip, and make a Notch or Mark there for the true and constant Dipping-place of that Back.

Then if any Quantity of Worts (which covers the whole Back) be deeped or gauged at that Place, and the wet Inches so taken be multiplied into the Area of the Back in Gallons, the Product will shew how many Gallons of Worts are in that Back at that time, provided the Sides of the Back do stand at Right-angles with the Bottom.



PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale or Wine Gallons.

IF the Square of the Diameter be multiplied by .002785 for Ale, or by .003399 for Wine; or if it be divided by 359.05 for Ale; or by 294.12 for Wine, the Products or Quotients will be the respective Ale or Wine Gallons.

Example. Suppose the Diameter of a Circle be 32.6 Inches; What will be the Area in Ale or Wine Gallons?

The Square of 32.6 is 1062.76.

Then $359.05 \mid 1062.76 (2.9599$ Area in Ale Gallons.
And $294.12 \mid 1062.76 (3.6133$ Area in Wine Gallons.
Or $1062.76 \times .002785 = 2.9598$ Ale Gallons.
And $1062.76 \times .0033993 = 3.6133$ Wine Gallons.

By

By Scale and Compasses.

Extend the Compasses from 18.95 (the Gauge-point for Ale) to 32.6 (the Diameter) that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compasses from 1 to 32.6, that Extent, turned twice over from .002785, will at last fall upon 2.9599.

For Wine extend from 17.15 (the Gauge-point for Wine) that Extent, turned twice over from 1, will at last fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003299, being turned twice over, to 3.6133 Wine Gallons.



PROBLEM IV.

The Transverse (or longest Diameter) and the Conjugate (or shortest Diameter) of an Ellipsis (or Oval) being given, to find its Area in Ale or Wine Gallons.

IF the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiplied by .002785 for Ale, or divided by 294.12, or multiplied by .003399 for Wine, the Quotient or Product will be the Ale or Wine Gallons required.

Example. Suppose the longest Diameter be 81.4 Inches, and the shortest Diameter be 54.6 Inches; What will be the Area of that Oval?

E c

Multiply

Multiply 81.4 by 54.6, and the Product is 4444.44; then,

359.05)4444.44(12.38 Area in Ale Gallons.

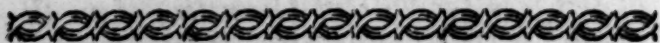
294.12)4444.44(15.11 Area in Wine Gallons.

Or $4444.44 \times .002785 = 12.38$ Ale Gallons.

And $4444.44 \times .003399 = 15.11$ Wine Gallons.

By Scale and Compasses.

First, find a mean Proportional between 81.4 and 54.6, by dividing the Distance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval.) Then extend the Compasses from 18.95 (the Gauge-point for Ale) to 66.6, that Extent, turned twice over from 1, will at last fall upon 12.38, Ale Gallons: And extended from 17.15 (the Gauge-point for Wine) to 66.6; that Extent, turned twice over from 1, will reach at last to 15.11 Wine Gallons.



PROBLEM V.

To find the Content in Ale or Wine Gallons of any Prism, whatsoever Form its Base is of.

FIRST, find its solid Content in Inches (by Sect. 1, 2, 3. of Chap. 2. Part II.) then divide that Content in Inches by 282 for Ale, or by 231 for Wine; the respective Quotients will be the Content in Wine or Ale Gallons.

Otherwise, you may find the Content of a Prism by finding the Area of its Base in Gallons (by Problem II. of this Appendix) and multiply that Area

2

by

Sect. I. Of Gauging.

313

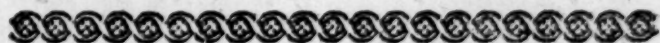
by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

Example. Suppose a Tun, whose Base is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale and Wine Gallons is required.

The Length, Breadth, and Depth, being multiplied continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons: And by dividing by 2150.4 such a Cistern will be found to hold 35.65 Bushels of Corn.

By Scale and Compasses.

Extend the Compasses from 282 to 36.5, the Breadth of the Base, that Extent will reach from 49.3, its Length, to 6.38 Ale Gallons, the Area of the Base; then extend from 1 to 42.6, the Depth, that Extent will reach from 6.38, the Area of the Base, to 271.8 Gallons the Content.



PROBLEM VI.

To find the Content of a Tun, whose Bases are alike and parallel, but unequal, being the Frustum of a Pyramid.

FIND the Area of each Base, and a mean Proportional between them, and multiply the Sum of those three by one third Part of the Depth or Height, and the Product is the Content.

E c 2

Example.

Example. Suppose a Tun, whose Bases are Parallelograms; the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the lesser Base 80, and its Breadth 56; and the Depth of the Tun 42 Inches; the Content in Ale and Wine Gallons is required.

Multiply 100 by 70, the Product is 7000, the Area of the greater Base; and 80 multiplied by 56, the Product is 4480, the Area of the lesser Base; then multiply the two Areas into each other; and the Product is 31360000, whose Square Root is 5600, a geometrical mean Proportional.

| | | |
|-----------------------|------|--------|
| The greater Area | 7000 | } add. |
| The lesser Area | 4480 | |
| The mean Proportional | 5600 | |

17080

A third of the Depth 14

68320

17080

282)239120(847.94 A. g.

231)239120(1035 15 W. g.

PROBLEM.

PROBLEM VII.

To find the Content of a Tun, whose Bases are parallel and circular, being the Frustum of a Cone.

YOU may find the Content as in the last Problem, by multiplying the Sum of the Areas of the two Bases, and a mean Proportional, by one third Part of the Depth.

But it will be a shorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and lesser Diameters add one third Part of the Square of the Difference of the Diameters; that Sum is the Square of a mean Diameter, which, divided by 359.05 for Ale, or by 294.12 for Wine, gives the Area of a mean Circle in Ale or Wine Gallons, which, multiplied by the Depth, gives the Content.

Example. Suppose the greater Diameter 80 Inches, and the lesser Diameter 71 Inches, and the Depth 34 Inches, the Content in Ale or Wine Gallons is required.

Multiply 80 by 71, and the Product is 5680; to which add 27 (a third Part of the Square of the Difference of the Diameters) and the Sum is 5707, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Gallons the Area; which multiply by 34 (the Depth), and the Product will be 540.43 Gallons, the Content.

By Scale and Compasses.

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter (though it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great); extend the Compasses from 18.95 (the Gauge-point for Ale) to 75.5, the mean Diameter; that Extent will reach from that 34 (the Depth) to a fourth Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compasses from 17.15 (the Gauge-point for Wine) to 75.5, that Extent will reach from 34, twice turned over, to 659.7 Gallons of Wine.

The Method used by the Gaugers for all such Tuns, is to take the Diameter in the Middle of every 10 Inches; that is, at five Inches from the Bottom, and at 15, and at 25, &c.

Then they find the Area to every one of these Diameters, and enter them in their Books. Then, when they survey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every 10 Inches they take the respective Areas, and remove the separating Point one Place towards the Right-hand; and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and so add all the several Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppose the Diameter at 5 Inches from the Bottom 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area answering to 64 Inches is 11.4078 Gallons; and to 67 Inches, is 12.5023 Gallons; and the Area to 70 Inches, is 13.647 Gallons; and to 73, is 14.8418 Gallons: Then, supposing the Depth of the Liquor in the said Tun be found

found to be 3.6 Inches: Now, to cast up this Gauge, first, in the Area answering to 64 Inches, being multiplied by 10, that is by removing the separating Point a Place towards the Right hand, it will be 114.078 Gallons; and the next will be 125.023; and the next 136.47 Gallons. Now these three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8418 by 3.6, and the Product is 53.4305: Add all together, and the Sum is the whole Quantity of Liquor in the Tun.

| | |
|-----------------------------------|---------|
| The Content at 10 Inches deep | 114.078 |
| The Content at the next 10 Inches | 125.023 |
| The Content at the next 10 Inches | 136.470 |
| The Content of the 3.6 Inches | 53.430 |

| | | |
|--------------------------------------------|-----------|-----------|
| The whole Quantity of Liquor in the Tun | — — — — — | } 429 001 |
|--------------------------------------------|-----------|-----------|



PROBLEM VIII.

To find the Drip or Fall of a Tun.

Suppose the Tun last mentioned was so placed, that when the Bottom is but just covered on one Side, the Liquor is 4 Inches deep on the Side opposite; How much must be allowed for the Fall of this Tun? That is, How much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area answering thereunto is 10.568; which multiplied by 2 (that is, half 4), the Product is 21.136 Gallons; and so much Liquor will just cover the Bottom.

But,

But, suppose it was set so much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the opposite Side just cuts between the Bottoms and Staves; How much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the Top Diameter, and divide the last Product by the Sum of the Diameters, and to the Quotient add the Square of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine; multiply the Quotient by the Depth, the Product is the Content.

The Bottom Diameter of the fore-mentioned Tun is 61 Inches; and the Diameter, at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61 is 3721; which multiplied by 71.5, the Product is 266051.5; this divided by 132.5 (the Sum of the Diameters) the Quotient is 2007.936: To which add 3721 (the Square of 61), and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiplied by 30, the Depth, the Product is 159.558, the Gallons of Liquor in the Tun.

When the Frustum of a Cone or Pyramid is cut, by a diagonal Plane, through the Extremities of the Diameters, as the Liquor in the Tun, represents, such Solid is called a Hoof. (Vide *Ward's Young Mathematician's Guide*, Page 414.)

If it be the Hoof of a square Frustum, instead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine. All the rest of the Work is the same.

PROBLEM IX.

To gauge a Copper.

LET ABCD be a small Copper to be gauged.

Take a small Cord of Packthread, make one End fast at A, and extend the other to the opposite Side of the Copper at B, where make it fast, or cause some Person to hold it very strait; then set one End of the Instrument in the Bottom of the Copper at C, and move it to and fro, till you find the nearest Distance to the Thread (at *a*): This Distance, *a* C, is the Depth of the Copper, which suppose to be 47 Inches.



In like manner, set the End of the Rule upon the Top of the Crown at *d*, and take the nearest Distance to the Thread, as *d* g, which suppose 42 Inches: This subtracted from *a* C, 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown.

Measure AB, the Diameter of the Top, which admit to be 99 Inches; then hold a Thread so as a Plummēt at the End thereof may hang just over C,
by

by which means you will find the Distance Aa . Do the like on the other Side; so will you find also the Distance, B ; which suppose 17.5 Inches each; add these two together, and subtract their Sum (*viz.* 35) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown. The Diameter which touches the Top of the Crown, may be found by the Sliding-rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards, that is, the Part $ABkb$, the Depth gd being 42 Inches, you may take the Diameter in the Middle of every 6 Inches of the Depth, which suppose to be as in the second Column of the following Table, the Numbers in the third Column are the respective Areas in Ale Gallons, found by Problem III. the fourth Column shews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, $ABkb$; that is, so much as it will hold after the Crown is covered.

Now, if the Crown be taken for the Frustum of a Sphere, the Content (by the latter Part of Sect. II. Page 190.) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus;

The Diameter CD was found to be 64, and the Area to this Diameter is 11.408; this, multiplied by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Gallons, the Content of the Crown.

The Content of the Part $bkDC$ is 57.935 Gallons; from which subtract the Content of the Crown, 28.52, and the Remainder is 29.415 Gallons, and so much Liquor will just cover the Crown.

| Parts of the Depth. | Diameter. | Areas. | Content of every 6 Inches. |
|---------------------------|-----------|---------|----------------------------------|
| 6 | 95.3 | 25.2945 | 151.767 |
| 6 | 90.1 | 22.6095 | 135.657 |
| 6 | 85.0 | 20.1223 | 120.734 |
| 6 | 80 | 17.8246 | 106.947 |
| 6 | 75.2 | 15.7499 | 94.499 |
| 6 | 70.5 | 13.8426 | 83.056 |
| 6 | 66 | 12.1319 | 72.791 |
| The Sum — — — | | | 765.451 |
| To just cover the Crown — | | | 29.415 |
| The whole Content — — — | | | 794.866 |

By Scale and Compasses.

You may find the Areas answering to every one of the Diameters, thus :

Extend the Compasses from the Gauge-point to the Diameter; that Extent being turned twice over from 1, will at last fall upon the Area of that Circle: Or, being turned twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compasses from 18.95 (the Gauge-point) to 95.3; that Extent, turned twice over from 6, will at last fall upon 151.76 Gallons, the Content of the first 6 Inches. And so of the rest.

PROBLEM X.

To compute the Content of any close Cask.

IN order to perform this difficult Part of Gauging, the three following Dimensions of the Cask must be truly taken;

Viz. { The Bung-diameter,
The Head-diameter,
The Length of the Cask, } within the Cask.

In taking these Dimensions, it must be carefully observed,

1. That the Bung-hole be in the Middle of the Cask; also, that the Bung-staff, and the Staff opposite to the Bung-hole, are both regular and even within.

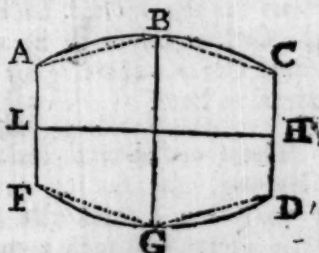
2. That the Heads of the Cask are equal, and truly circular; if so, the Distance between the Inside of the Chine to the Outside of its opposite Staff, will be the Head-diameter within the Cask, very near.

3. With a sliding Pair of Calipers (made for that Use) take the shortest Distance, or Length, between the Out-sides of the two Heads; from that Length subtract $1\frac{1}{2}$ Inch (more or less according to the Largeness of the Cask) for the Thickness of the Head: The Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length, by putting a strait Rod in at the Tap-hole, and allow for the Thickness of the Head.

Now by these Dimensions, one would think the Content of the Cask was perfectly limited; but it will be easy to perceive, by the following Figure, that
the

the Diameters and Length of one Cask may be equal to those of another, and yet one of those Casks may contain several Gallons more than the other.

As for Instance, the Figure ABCDF is supposed to represent a Cask: Then it is plain, that if the outward curve Lines, ABC, and FGD, are the Bounds or Staves of the Cask, it must needs hold more than if the inner pricked Lines were the Bounds, or Staves; and yet the Bung-diameter BG, and Head-diameters CD and AF, and the Length LH, are the same in both those Casks.



Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Casks can be gauged: And therefore Gaugers do usually suppose every Cask to be in some of these Forms:

1. The middle Frustum of a Spheroid.
2. The middle Frustum of a parabolic Spindle.
3. The lower Frustums of two equal parabolic Conoids.
4. The lower Frustums of two equal Cones.

1. If the Staves of a Cask be very much curved (as the outward Lines of the last Figure), then the Cask is supposed to be the middle Frustum of a Spheroid.

2. If the Staves (between the Bung and the Head) be something less curved, then the Cask is taken to be the middle Frustum of a parabolic Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Frustums of two equal parabolic Conoids,

F f

abutting

abutting or joining together upon one common Base.

4. If the Staves between the Bung and Head be strait (as the pricked Lines in the last Figure), then the Cask is taken to be the lower Frustums of two equal Cones, abutting or joining together upon one common Base.

There are several Rules laid down in Books of Gauging, for finding the Content of each several Form; but I think the shortest and most practical Way is, to find such a mean Diameter, which will reduce the proposed Cask to a Cylinder: Thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the second Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-diameter, and the Sum is the mean Diameter.

Example. Suppose the Bung-diameter be 32 Inches, the Head-diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is required.

The Difference between the Bung and Head diameter is 8; which multiplied by .7, the Product is 5.6; which added to the Head-diameter, the Sum is 29.6, the mean Diameter: The Area answering thereunto will be found (by Prob. III.) to be 2.44 Ale Gallons; which multiplied by the Length, the Product is 97.4 Gallons; and so much is the Content, if it be the first Form.

Again; if the Difference of the Diameters 8 be multiplied by .65, the Product will be 5.2; which added to the Head-diameter, the Sum is 29.2, for the mean Diameter; and the Area answering thereunto is 2 3/4 Gallons; which multiplied by 40 (the Length) the Product is 94.98 Gallons, the Content, if it be of the second Form.

Again; if the Difference 8 be multiplied by 6, the Product is 48; which added to the Head-diameter, the Sum is 28.8, the mean Diameter: The Area thereunto is 2.31 Gallons; which, multiplied by 40, gives the Content 92.4 Gallons, for the third Form.

Again; the Difference 8, multiplied by .55, the Product is 4.4; which added to the Head-diameter, makes the mean Diameter 28.4; the Area thereof is 2.2463; which multiplied by 40, the Product is 89.85 Gallons, for the fourth Form.

By Scale and Compasses.

Extend the Compasses from the Gauge-point 18.95 to the first mean Diameter 29.6; that Extent will reach from the Length 40 to a fourth Number, and then to the Content, 97.4 Gallons.

Again; extend from 18.95 to 29.2 (the second mean Diameter) that Extent, turned twice over from 40, will at last fall upon 94.98 Gallons.

Again; extend from 18.95 to 28.8 (the third mean Diameter) that Extent, turned twice over from 40, will at last fall upon 92.4 Gallons.

Again; extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turned twice over from 40, will at last fall upon 89.85 Gallons.

Although I have all along made use of the Line of Numbers upon the common Two foot or Eighteen-inch Rules, for the Reason mentioned in the *Preface*; yet the Rules may easily be applied to the Sliding-rule, thus: To find the Area of a Circle in Gallons, set the Gauge-point upon D (that is, a single Line of Numbers) to 1 upon C (that is, a double Line;)

F f 2

then

then against any Diameter upon D, is the Area upon C, thus;

To find the Content of the Cask, last-mentioned, the first Form.

Set the Gauge-point 18.95 upon D, to the Length 40 upon C; then (against the mean Diameter) 29.6 upon D, is 97.4 Gallons, the Content upon C.

And against 29.2 (the next mean Diameter) on D, is 94.98 Gallons on C.

And against 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And against 28.4 (the last mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE of the Segment of a Circle,
whose Area is Unity.

| V. S. | Segm. | V. S. | Segm. | V. S. | Segm. | V. S. | Segm. |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | .0017 | 99 | .9983 | 26 | .2066 | 74 | .7924 |
| 2 | .0048 | 98 | .9952 | 27 | .2178 | 73 | .7832 |
| 3 | .0087 | 97 | .9913 | 28 | .2292 | 72 | .7708 |
| 4 | .0134 | 96 | .9866 | 29 | .2407 | 71 | .7593 |
| 5 | .0187 | 95 | .9813 | 30 | .2523 | 70 | .7477 |
| 6 | .0245 | 94 | .9755 | 31 | .2640 | 69 | .7360 |
| 7 | .0308 | 93 | .9692 | 32 | .2759 | 68 | .7241 |
| 8 | .0375 | 92 | .9625 | 33 | .2878 | 67 | .7122 |
| 9 | .0446 | 91 | .9554 | 34 | .2998 | 66 | .7002 |
| 10 | .0520 | 90 | .9480 | 35 | .3119 | 65 | .6881 |
| 11 | .0598 | 89 | .9402 | 36 | .3241 | 64 | .6750 |
| 12 | .0680 | 88 | .9320 | 37 | .3364 | 63 | .6636 |
| 13 | .0764 | 87 | .9236 | 38 | .3487 | 62 | .6513 |
| 14 | .0851 | 86 | .9149 | 39 | .3611 | 61 | .6389 |
| 15 | .0941 | 85 | .9059 | 40 | .3735 | 60 | .6265 |
| 16 | .1033 | 84 | .8967 | 41 | .3860 | 59 | .6140 |
| 17 | .1127 | 83 | .8873 | 42 | .3986 | 58 | .6014 |
| 18 | .1224 | 82 | .8776 | 43 | .4112 | 57 | .5888 |
| 19 | .1323 | 81 | .8677 | 44 | .4238 | 56 | .5762 |
| 20 | .1424 | 80 | .8576 | 45 | .4364 | 55 | .5636 |
| 21 | .1526 | 79 | .8474 | 46 | .4491 | 54 | .5509 |
| 22 | .1631 | 78 | .8369 | 47 | .4618 | 53 | .5382 |
| 23 | .1737 | 77 | .8263 | 48 | .4745 | 52 | .5250 |
| 24 | .1845 | 76 | .8155 | 49 | .4873 | 51 | .5127 |
| 25 | .1955 | 75 | .8045 | 50 | .5000 | 50 | .5000 |

The Use of the Table of Segments

Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whose Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.

The R U L E is;

To the wet or dry Inches of the Bung-diameter, add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiplied by the whole Content of the Cask, the Product shews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Cask in Form of a Cylinder, whose Bung-diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Cask?

Divide the dry Inches 13, by 29 the Bung-diameter, and the Quotient is .448; find the two first Figures .44 under V. S. and the Segment against it is .4238; to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiplied by the Content of the Cask, the Product will be 34.664 Gallons; and so much the Cask wants of being full.

Note, If the Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a spheroidal Cask, then use the following Method.

1. By

1. By the Bung and Head-diameter, find such a mean Diameter as, you judge, will reduce the proposed Cask to a Cylinder, and then find its Content.

2. From the Bung-diameter subtract the mean Diameter, and take half the Difference.

3. From the wet Inches subtract the said Half-difference; reserve this Difference, then use the Proportion:

As the mean Diameter is to 100
(the Diameter of the tabular Circle),
So is the reserv'd Difference
to a versed Sine in the Table.

Then, if the tabular Segment be multiplied into the Content (as before) the Product will be the Quantity of Liquor in the Cask.

Example. Let the Cask be the same as in Page 325, of the first Form, where the Bung-diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and suppose the wet Inches 19, to find the Quantity of Liquor in the Cask.

From 32
Subtr. 29.6

Rem. 2.4

Half 1.2

From 19
Subtr. 1.2

Rem. 17.8 reserved.

29.6 : 100 :: 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiplied by 97.4, the Content, the Product is 61 Gallons, the Quantity of Liquor in the Cask.

If the dry Inches had been given, by the same Method, you might have found the Ullage, or what the Cask wanted of being full.

To find what Quantity of Liquor is in a Cask, when its Axis is perpendicular to the Horizon; *viz.* when it stands upright upon one of its Heads.

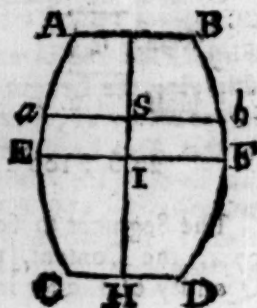
To do this, you must know how to calculate the Area of any Circle, between the Bung and Head, whose Distance from the Bung, or Middle of the Cask, is given; which may be done by this Proportion.

As the Square of half the Length of the Cask is to the Difference between the Bung and Head-areas; so is the Square of any Circle's Distance from the Bung, to the Difference between the Bung-area and the Area of that Circle; *viz.* the Area of the Liquor's Surface.

Then, from the Bung-area, subtract one-third Part of the aforesaid Difference; *viz.* between the Bung-area and the Area of the Liquor's Surface: Multiply the Remainder by the Liquor's Distance from the Bung, and the Product will shew what Quantity of Liquor is either above or under half the Content of the Cask.

Example. Let us again suppose the Cask, in Page 325, whose Length is 40 Inches, Bung-diameter 32, and Head-diameter 24, and suppose the wet Inches, SH, 26 Inches.

The Square of half the Length is 400, the Distance of the Liquor's Surface from the Bung SI is 6, whose Square is 36; the Area of the Bung D 2.85¹/₉ Ale Gallons, and the



Sect. I. *Of Gauging.* 333
 the Area of the Head D 1.6042; the Difference
 1.2477. Then,

$$400 : 1.2477 :: 36 : .0751$$

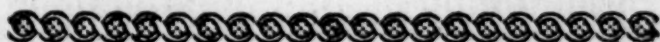
One-third is = .0250

From 2.8519 Bung-area,
 Subtr. .0250 a Third of the Difference.

Rem. 2.8269
 6 mult. Distance from the Bung

16.9614 Content above the Bung.
 Add 48.7 half the Content of the Cask.

65.66 the Quantity of Liquor in the
 [Cask,



PROBLEM IX.

Gauging of MALT.

TO find the Quantity of Malt in a Cistern, or
 upon a Floor.

First, Find the Area of the Base in Bushels, by
 multiplying the Length by the Breadth, and dividing
 the Product by 2150.42, or only by 2150; and mul-
 tiply that Area by the mean Depth (How to take the
 mean Depth, see Problem II.) If the Base be cir-
 cular or oval, divide by 2731 (see Problem I.)

Example

Example. There is a Cistern, whose Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Bushels, the Area of the Bottom at 1 Inch deep; which multiplied by the Depth 43.6, the Product is 91.98 Bushels, the Content.

Example. Suppose a Quantity of Malt upon a Floor, whose Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; How many Bushels are there?

Multiply 245 by 184, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Base; which multiplied by the mean Depth, the Product is 117.4 Bushels, the Content.

By the Sliding-Rule.

There is an inverted Line of Numbers upon some Sliding-Rules, marked with the Letter M, which was contrived purposely for Gauging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of these are of equal Radius, and all work together at once: Thus set the Length and Breadth against one another upon the inverted Line, and that which slides by it; then, on the other Edge of the Rule against the Depth, you will find the Content in Bushels. Thus, in the first Example, set 54 upon the Slider against 84 upon the inverted Line; and then, against 43.6 upon the other Part of the Rule, is 91.98 upon the Slider.

Again; in the second Example, set 184 upon the Slider to 245 upon the inverted Line; and against 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.



§ II. OF LAND-MEASURING.



SHALL not here give the whole Art of *Surveying*, but such practical Rules only as may be useful to the Country Grasers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only (and for want of that, with a Pole or Stick of half a Rod in Length.)



PROBLEM I.

To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or long Square, or what is something near that Form.

TO know whether any Angle in the Field be a Right angle, or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or square; and, with a Saw, cut two Kerfs, crossing each other at Right-angles; and bore a Hole in the Middle of the Back side, to put it upon the End of a Stick. This will represent the Instrument called a Cross.

Suppose

Suppose you would observe the Angle A, to know whether it be a Right-angle (or near thereunto); prick up your Stick, with the Cross upon it, a little Distance from the Fence, as at *a*; and having set up two Marks, as at *b* and *c*, of equal Distance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.



To measure such a Piece of Ground as this Figure above: If you measure round, and add the opposite Sides together, and take half the Sum (if they be not equal); or else measure down about the Middle of the Length, and Middle of the Breadth; thus, the Side AB being measured, it will be 5.60 (that is, 5 Chains and 60 Links); and the opposite Side CD is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10 38; and the Side AC 10.22; and the half Sum thereof is 10.30 (it will be the same Thing, if you measure about the Middle of the Length and Middle of the Breadth); then multiply this mean Length and mean Breadth together; viz. 10.30, by 5.71, and the Product is 58.8130; which divide by 10 (because 10 Square Chains is an Acre) by removing the separating Point one Place towards the Left.

Sect. II. Of Land-Measuring. 337

Left-hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 15 Places, and it will be 3.52520; which 3 towards the Left-hand are 3 Rods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left-hand are 21 Perches.

A. R. P.

So the whole Content is 5 3 21

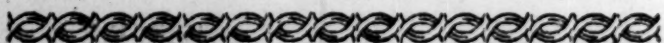
See the Work.

| | |
|----------|----------|
| 5.71 | |
| 10.30 | |
| 17130 | |
| 571 | A. R. P. |
| 5.88130 | 5 3 21 |
| 4 | |
| 3.52520 | |
| 40 | |
| 21 00800 | |
| | |

Note, The Chain here made use of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But, because every Man that may have Occasion to measure a Piece of Land, can't procure a Chain, I will therefore shew how you may measure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; which Stick divide into five equal Parts, so will the whole Rod be divided into ten Parts, and will thereby be adapted to Decimal Arithmetic.

But, because each of those Parts of the Stick are something large (each Part being 19 Inches and 8 Tenths) it will be necessary to take your Dimensions to half of one of those Parts; and then, for that half Part, set 5 in the Place of Seconds, thus, suppose 3 Parts and a half, set it down thus, .35.



PROBLEM II.

LET us suppose a Field in the Form of a long Square, whose Length is 45 Rods 5 Parts and a half, and the Breadth 31 Rods 4 Parts and a half; What is the Content?

Multiply the Length and Breadth together, and divide the Product by 160 (because 160 Square Rods are an Acre) and the Quotient is Acres.

$$\begin{array}{r}
 45 \cdot 55 \\
 31 \cdot 45 \\
 \hline
 22775 \\
 18220 \\
 \hline
 4555 \\
 13665 \\
 \hline
 1432 \cdot 5475
 \end{array}$$

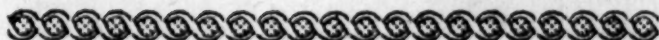
$$\begin{array}{r} 16 \overline{) 01432} (8 \\ \underline{128} \end{array}$$

A. R. P.

Facit 8 3 32

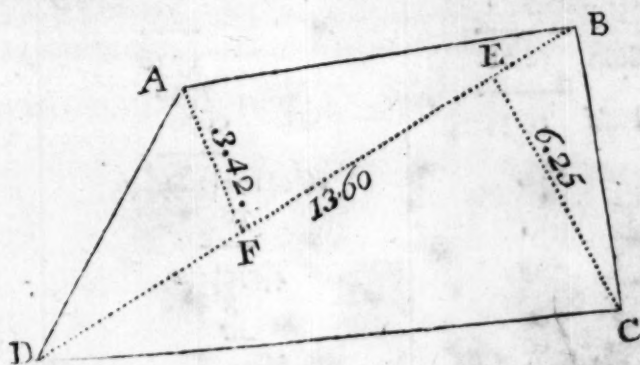
$$\begin{array}{r} 4 \overline{) 0152} (3 \\ \underline{12} \end{array}$$

32



PROBLEM III.

Suppose a Piece of Ground in the Form of a Trapezium; the Diagonal BD 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links; What is the Content;



Multiply the Diagonal by half the Sum of the Perpendiculars. See Sect. VI. of Chap. I. Part II.

340

CE = 6.25

AF = 3.42

Sum 9.67

Half 4.83

Appendix.

13 60 = BD

4.83

4080

10880

5440

6.56880

4

2.227520

40

11.00800

Sect. II.

A. R. P.

Facit 6 2 11

By Rods, thus;

CE = 25 Rods.

AF = 13.68

Sum 38.68

Half 19.34

19.34

544 = BD

7736

7736

9670

16|0)105|2.096(6

96

4|0)9|2(2

8

12

A. R. P.

Facit 6 2 12

To

To take the Dimensions of the Field.

Begin at the Angle B, and measure in a direct Line towards D; but when you come at E, set up your Cross, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then are you just in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards and forwards till it does so; then measure the Perpendicular, and set down the Chains and Links, or the Rods and Parts; then continue your Measure towards D; but when you come to F, set up your Cross, and try (as is above directed), whether you be in the Place where the Perpendicular will fall. Then measure the Perpendicular AF, and set down the Chain and Links, or Rods and Parts; then continue your Measure to D, and set down the Measure of the whole Diagonal. This Way of measuring is very exact and true; but the common Way used by the Graziers and Farmers, is to measure round the Field, and to take half the Sum of the opposite Sides for a mean Side; but the last-mentioned Piece of Ground, being measured so, will come to

A. R. P.

R. P.

7 0 22, which is 2 10 more than the Truth.

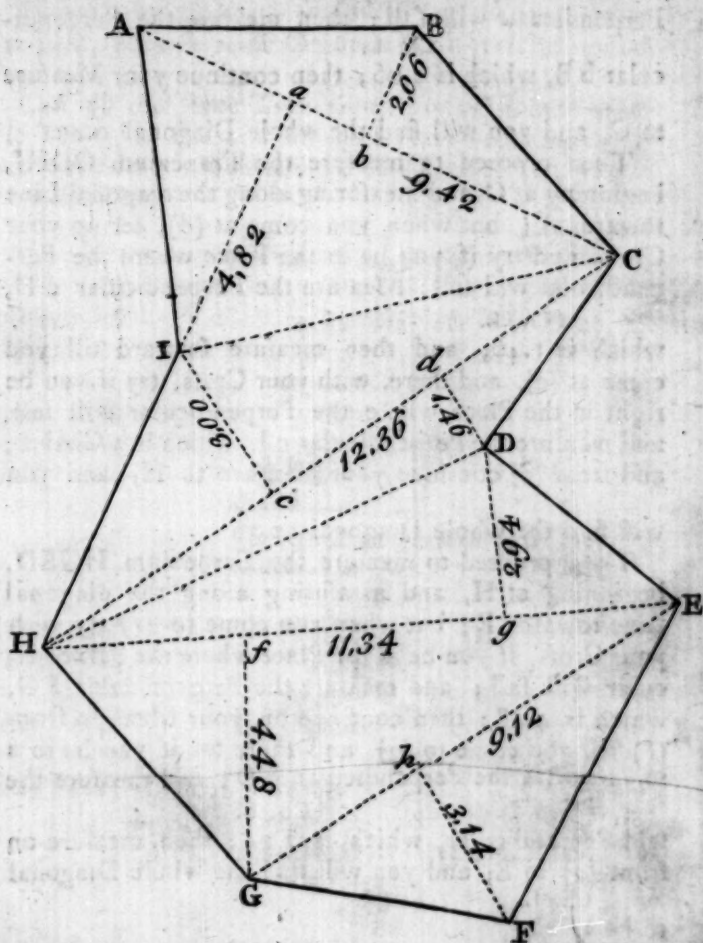
PROBLEM IV.

How to measure an irregular Field.

THE Way to measure irregular Land, is to divide it into Trapeziums and Triangles, thus :

First, view over the Field, and set up Marks at every Angle, and by those Marks you may see where to have a Trapezium, as ABCI in the following Figure.

Then



Then begin and measure in a direct Line from A towards C; but when you come to (a), set up your Cross, and try whether you be in a Square to I (as is before directed); and then measure the Perpendicular

Ch. L.

a I, which is 4.82; then measure forward again towards C, but when you come to (b) set up your Cross,

Cross, and try whether you be in the Place where the Perpendicular will fall; then measure the Perpendicular

Ch. L.

cular b B, which is 2.06; then continue your Measure

Ch. L.

to C, and you will find the whole Diagonal 9.42.

Then proceed to measure the Trapezium CDHI, beginning at C, and measuring along the diagonal Line towards A; but when you come at (d), set up your Cross, and try if you be in the Place where the Perpendicular will fall: Measure the Perpendicular d D,

Ch. L.

which is 1.46, and then measure forward till you come at (c), and there, with your Cross, try if you be right in the Place where the Perpendicular will fall, and measure the Perpendicular c I, which is 3 Chains; and from (c) continue your Measure to H, and you

Ch. L.

will find the whole Diagonal 12.36.

Then proceed to measure the Trapezium HGED, beginning at H, and measuring along the diagonal Line towards E; but when you come to (f) try with your Cross, if you be in the Place where the Perpendicular will fall; and measure the Perpendicular f G, which is 4.48; then continue on your Measure from (f) till you come to (g), and there try if you be in a Square with the Perpendicular g D; and measure the

Ch. L.

said Perpendicular, which is 2.94; then measure on from (g) to E, and you will find the whole Diagonal

Ch. L.

to be 11.34.

Then measure the Triangle EFG, beginning at E, and measuring along the Base EG, till you come at (h), and there with your Cross try if you be in the Place where the Perpendicular will fall; and mea-

Ch. L.

sure the Perpendicular h F, which is 3.14, continue your Measure to G, and you will find the whole Base

Ch. L.

to be 9.12; so you have finished your whole Field.

I have

Sect. II. *Of Land-Measuring.* 345

I have been the larger upon the Explanation of this Problem, because most Grounds lie in such irregular Forms.

Cast up the three Trapeziums severally, and also the Triangle; and add all the several Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

| | | |
|------------|---------|------------------------|
| b B = 2 06 | 9.42 | See Sect. VI. Chap. I. |
| a I = 4 82 | 3.44 | Part II. |
| <hr/> | <hr/> | |
| Sum 6.88 | 3768 | |
| <hr/> | 3768 | |
| Half 3 44 | 2826 | |
| | <hr/> | |
| | 3.24048 | = Area of ABCI. |
| | <hr/> | |

| | |
|------------|-----------------|
| d D = 1.46 | 12.36 |
| c I = 3.00 | 2.23 |
| <hr/> | <hr/> |
| Sum 4 46 | 3708 |
| <hr/> | 2472 |
| Half 2.23 | 2472 |
| | <hr/> |
| | 2.75628 |
| | <hr/> |
| | = Area of CIHD. |

| | |
|------------|-----------------|
| f G = 4 48 | 11.34 |
| g D = 2.94 | 3.71 |
| <hr/> | <hr/> |
| Sum 7.42 | 1134 |
| <hr/> | 7938 |
| Half 3.71 | 3402 |
| | <hr/> |
| | 4.20714 |
| | <hr/> |
| | = Area of HGED. |

Base

$$\text{Base} = 9.12$$

$$\text{Half} = 4.56$$

$$\text{Pepend.} = 3.14$$

$$1824$$

$$456$$

$$1368$$

$$1.43184 = \text{Area of the Trian. EFG.}$$

$$3.24048 = \text{Area of ABCI.}$$

$$2.75628 = \text{Area of CIHD.}$$

$$4.20714 = \text{Area of HGED.}$$

$$\text{Sum } 11.63574 = \text{Area of the Whole.}$$

$$4$$

$$2.54296$$

$$40$$

$$21.71840$$

A. R. P.

Facit 11 2 21

F I N I S.

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